

HERDER



Filling up a bottle

Name: **Timo Huber**

From **Berlin**

Team: Wir hätten lieber ausgeschlafen



„When a vertical water jet enters a bottle, sound may be produced, and, as the bottle is filled up, the properties of the sound change.

Investigate how *relevant parameters of the system* such as *speed* and *dimensions* of the jet, *size* and *shape* of the bottle or *water temperature* affect the sound.“



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relevant parameters



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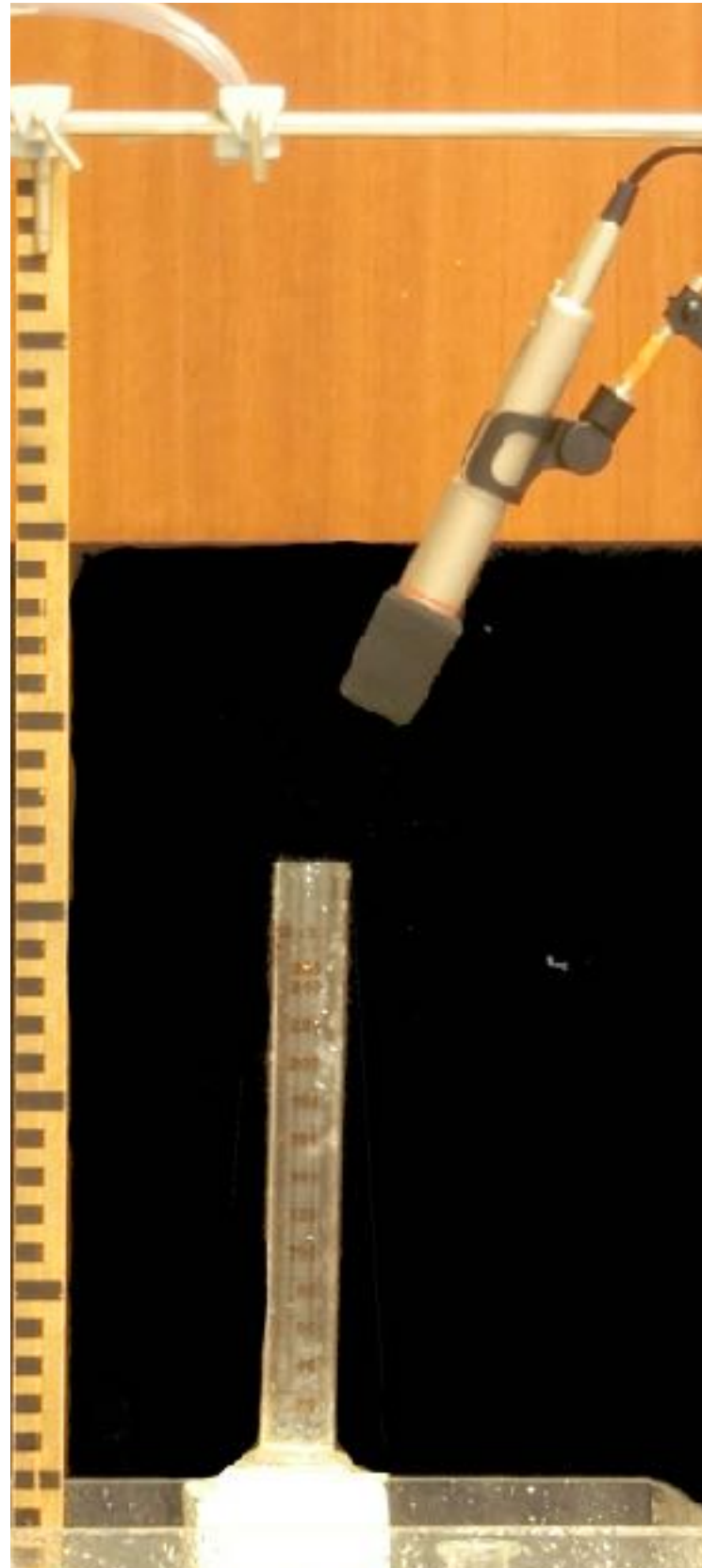


Demonstration

Experimental Setup



Water pumped from
overflowing container



Linear measuring
microphone

Audiointerface

Audio-analysis program

Audacity

VoceVista

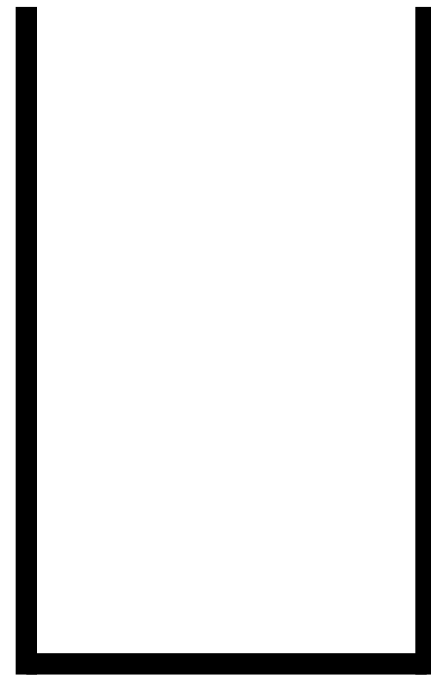
Video Pro

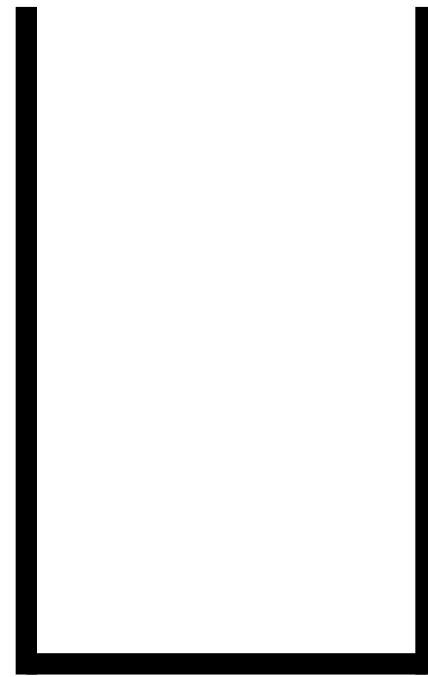
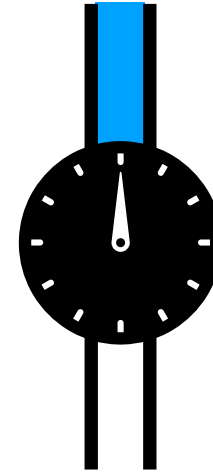
Camera films
from >5m with a
telephoto lens

Video-analysis program

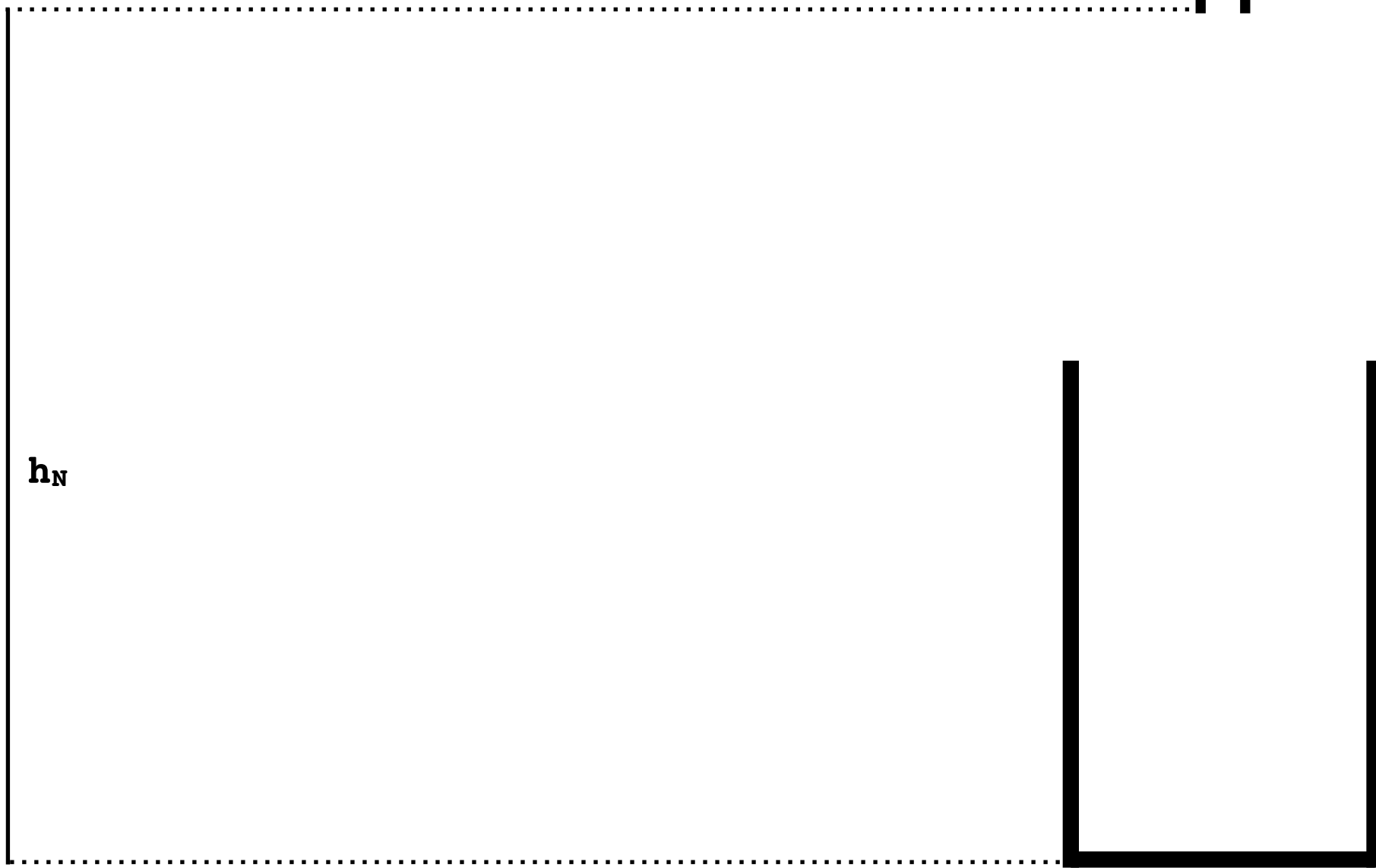
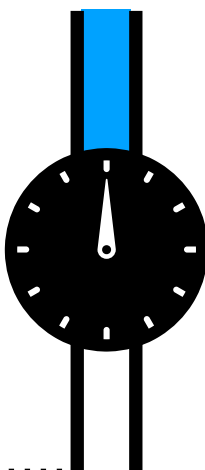
VianaNET







h_N : Nozzle height over container ground

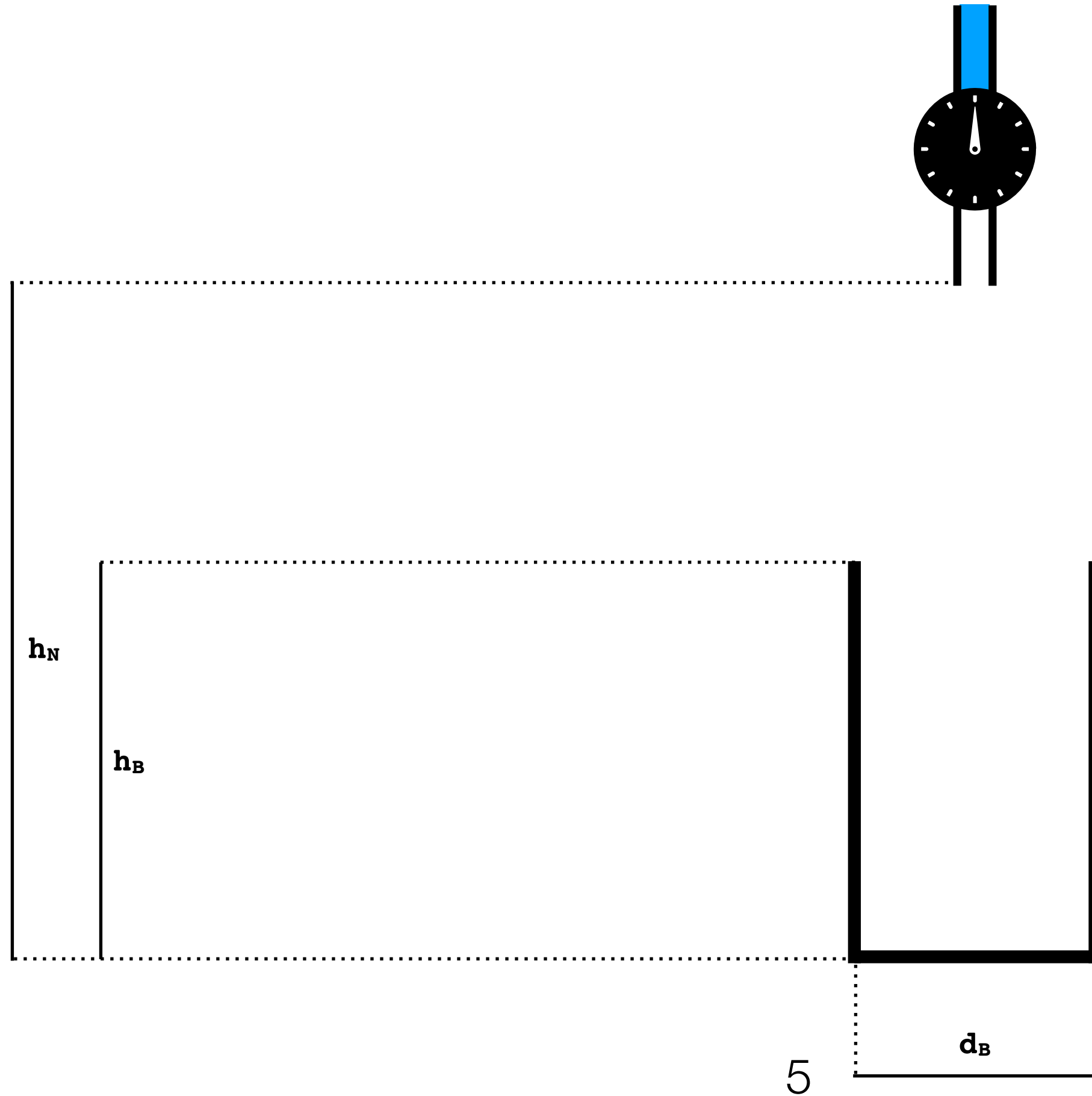




h_N : Nozzle height over container ground

h_B : Container-/ bottle-height

d_B : Container-/ bottle-diameter

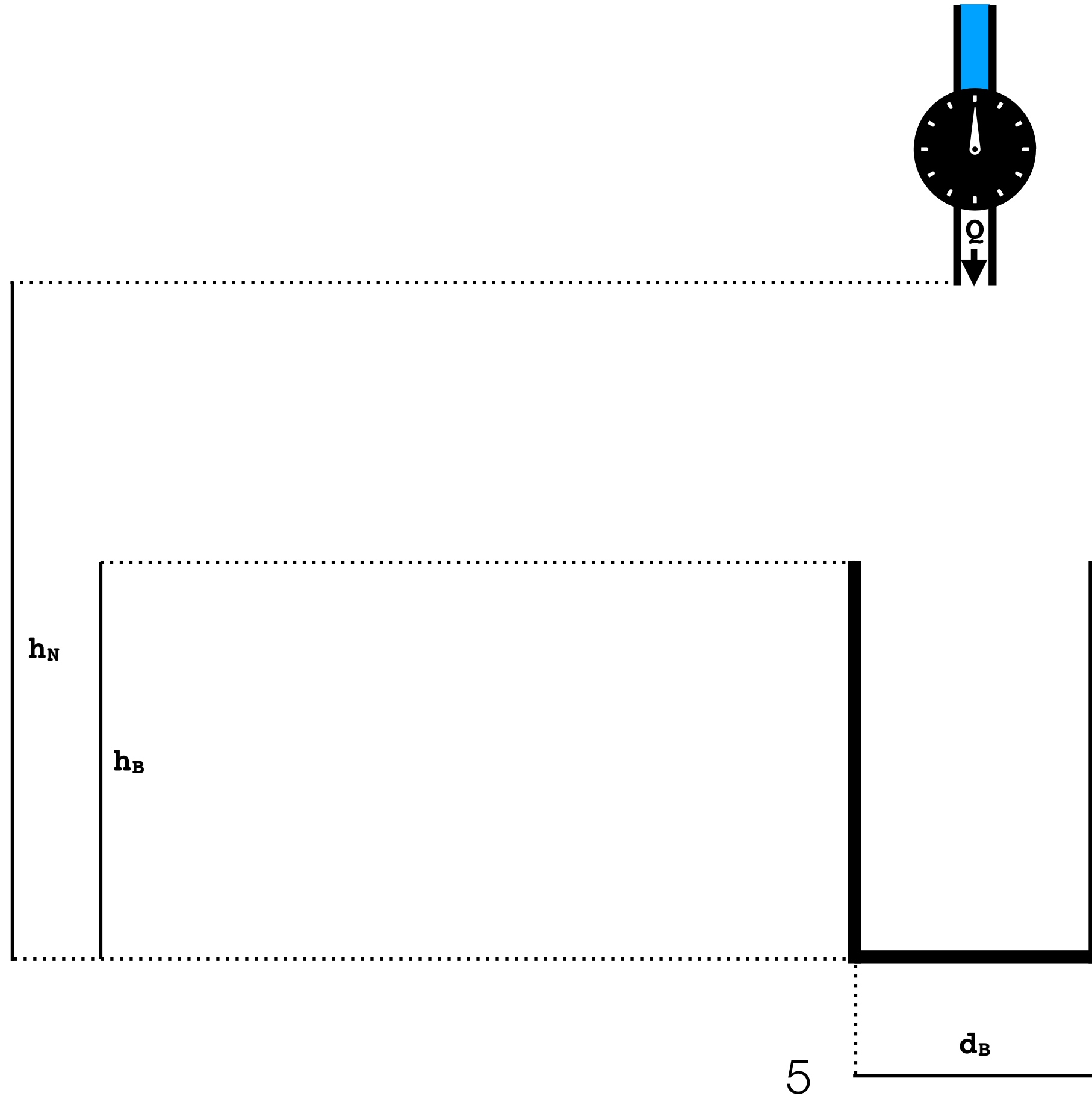




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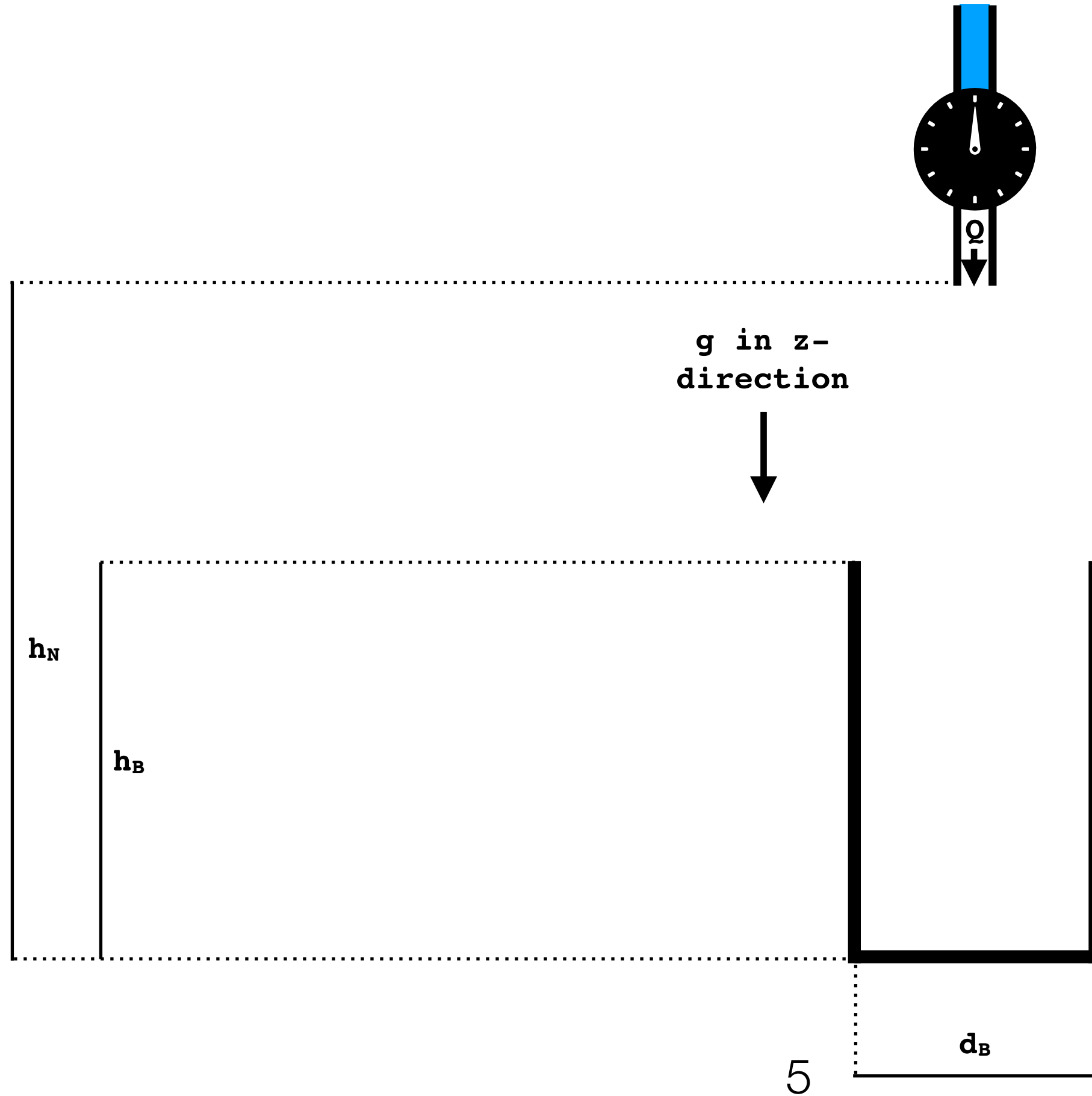




h_N : Nozzle height over container ground

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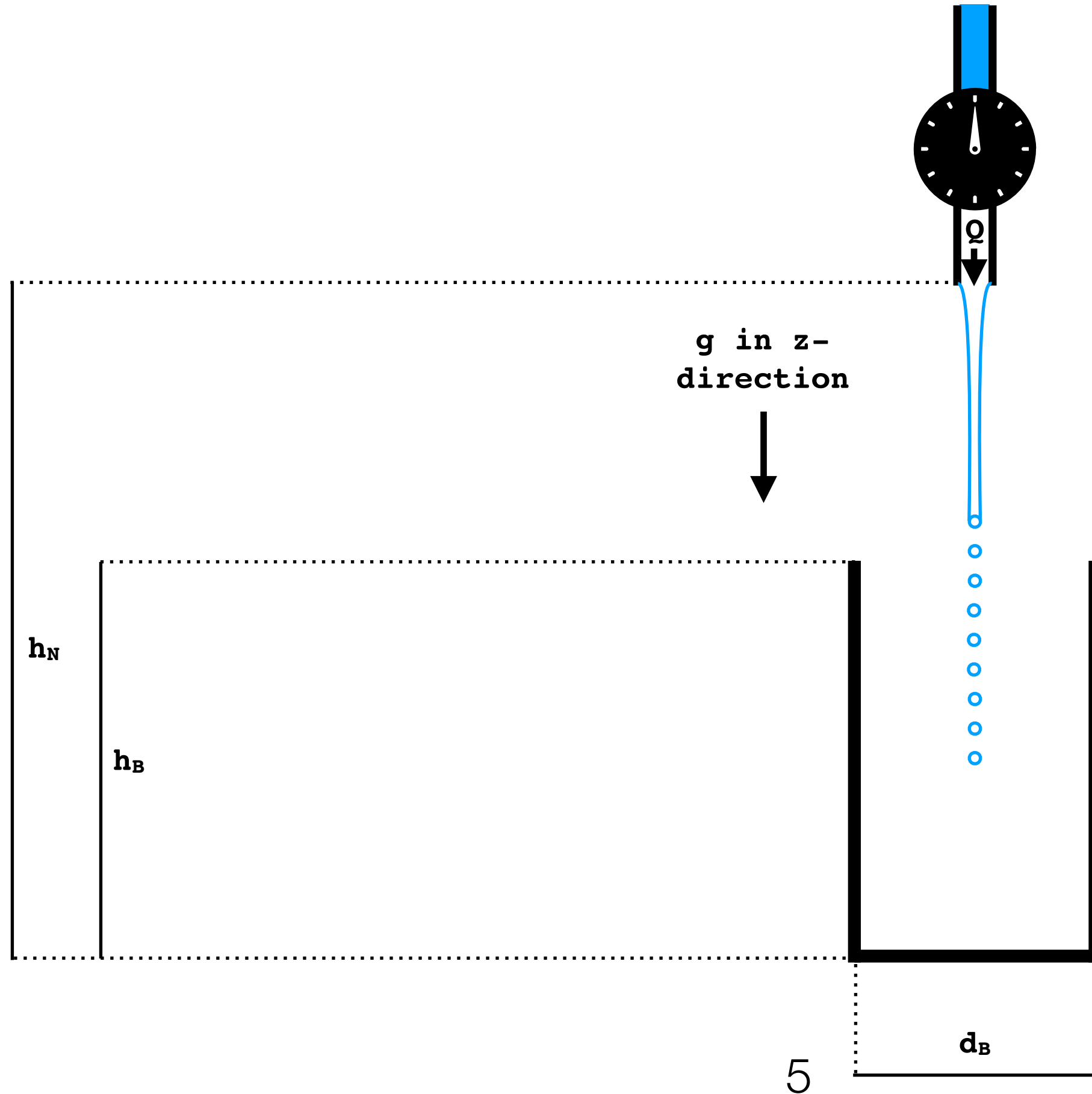
d_B : Container-/ bottle-diameter





Properties of the jet:
 Q : Flow rate
 r_o : Nozzle radius
 U_o : Speed at the nozzle

h_N : Nozzle height over container ground
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 d_B : Container-/ bottle-diameter

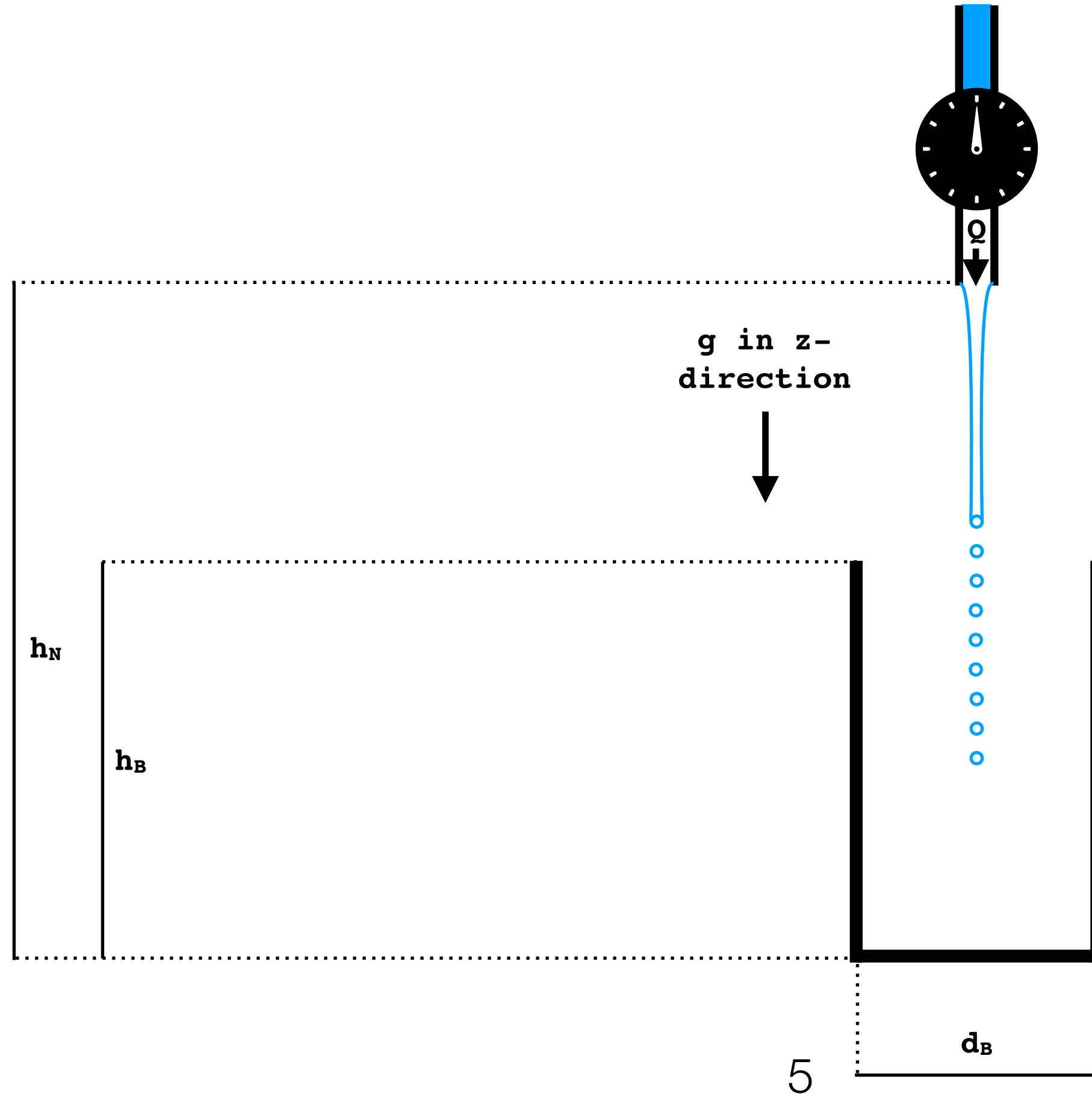




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Properties of water:
 ρ : Density
 σ : Surface tension
 η : dynamic viscosity
 T_w : Water temperature
(influences ρ, σ, η)



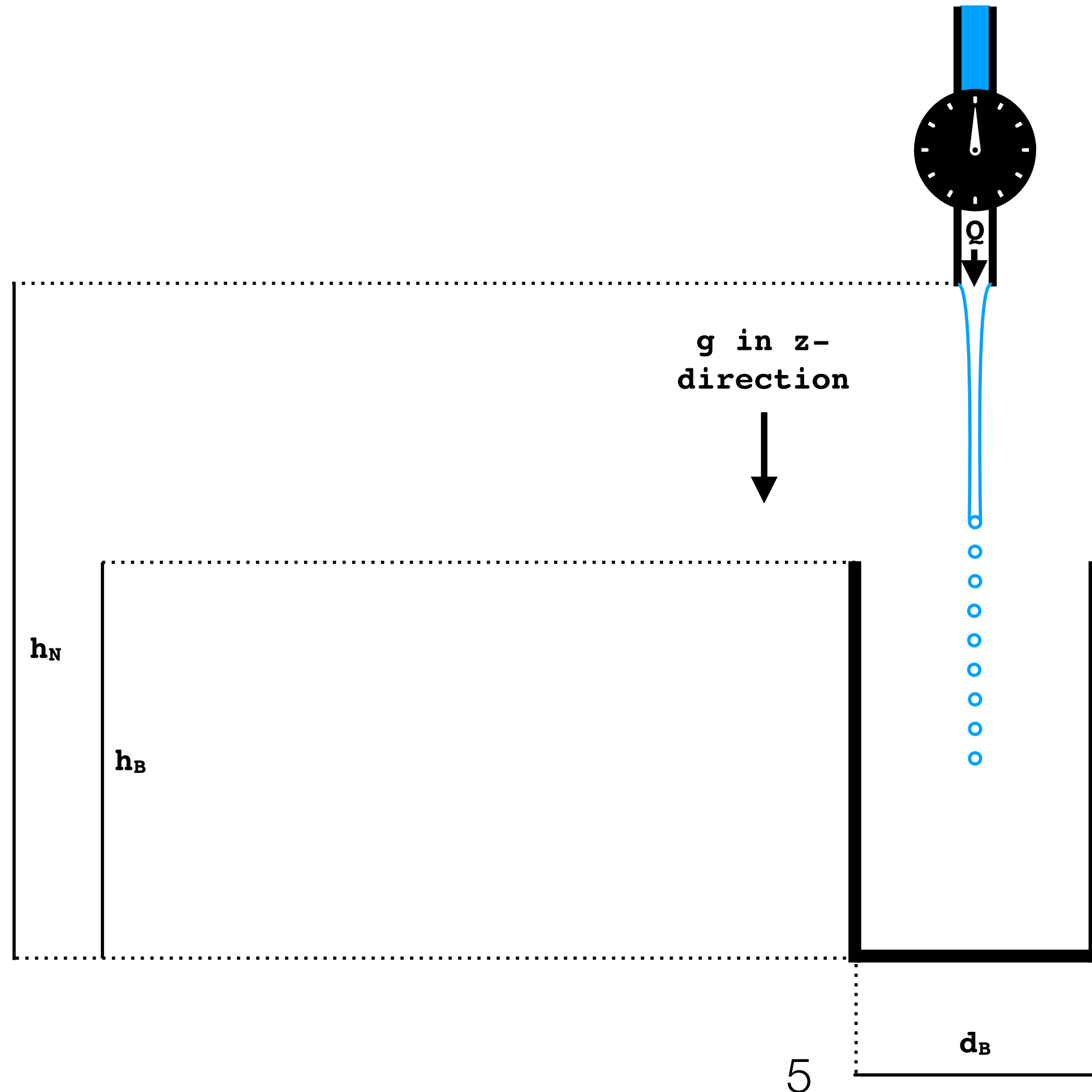


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Properties of air:
 p_o : Pressure
 γ : Adiabatic coefficient
 c : Speed of sound
 T_A : Air temperature
(influences p_o, c)



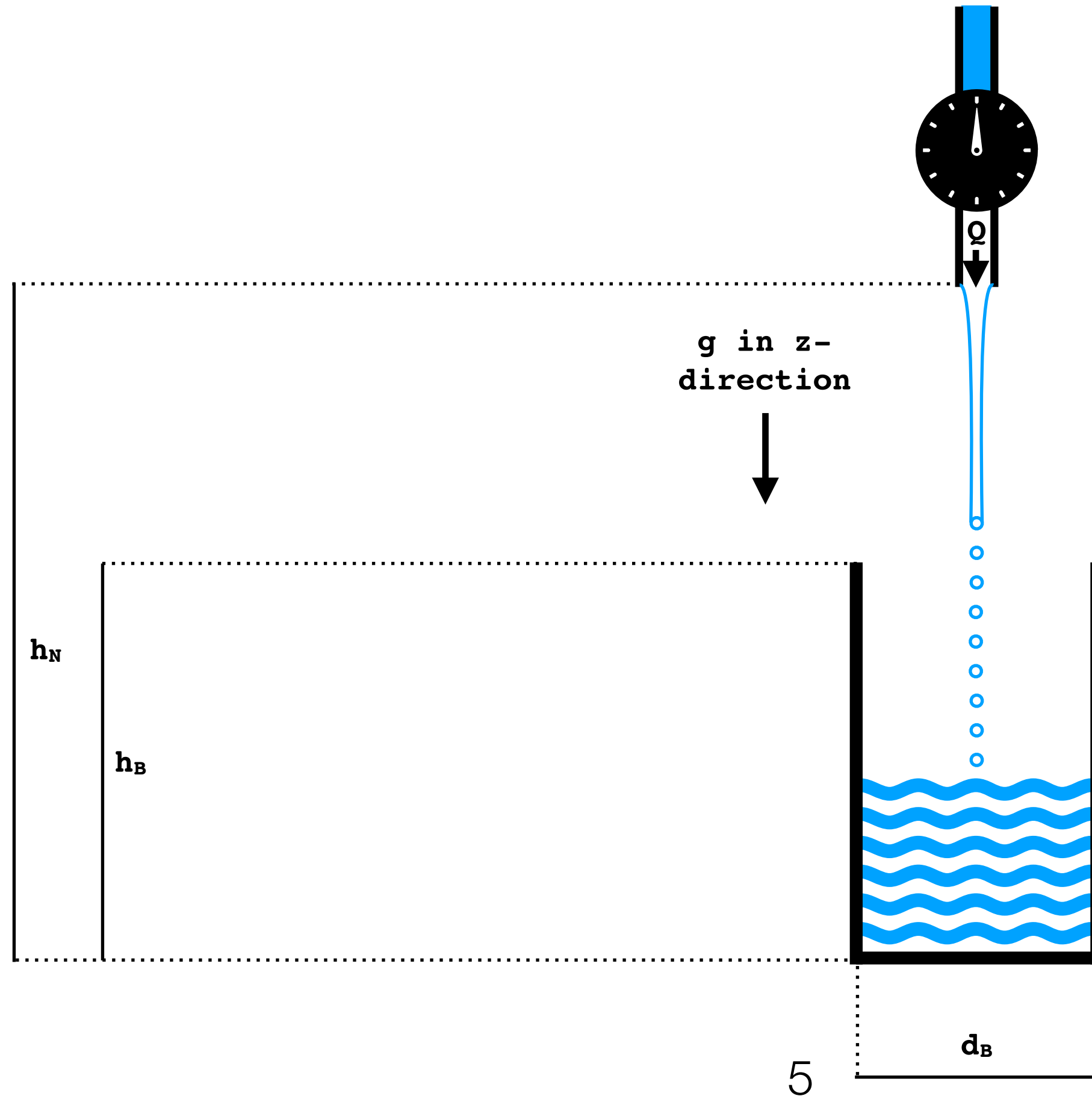


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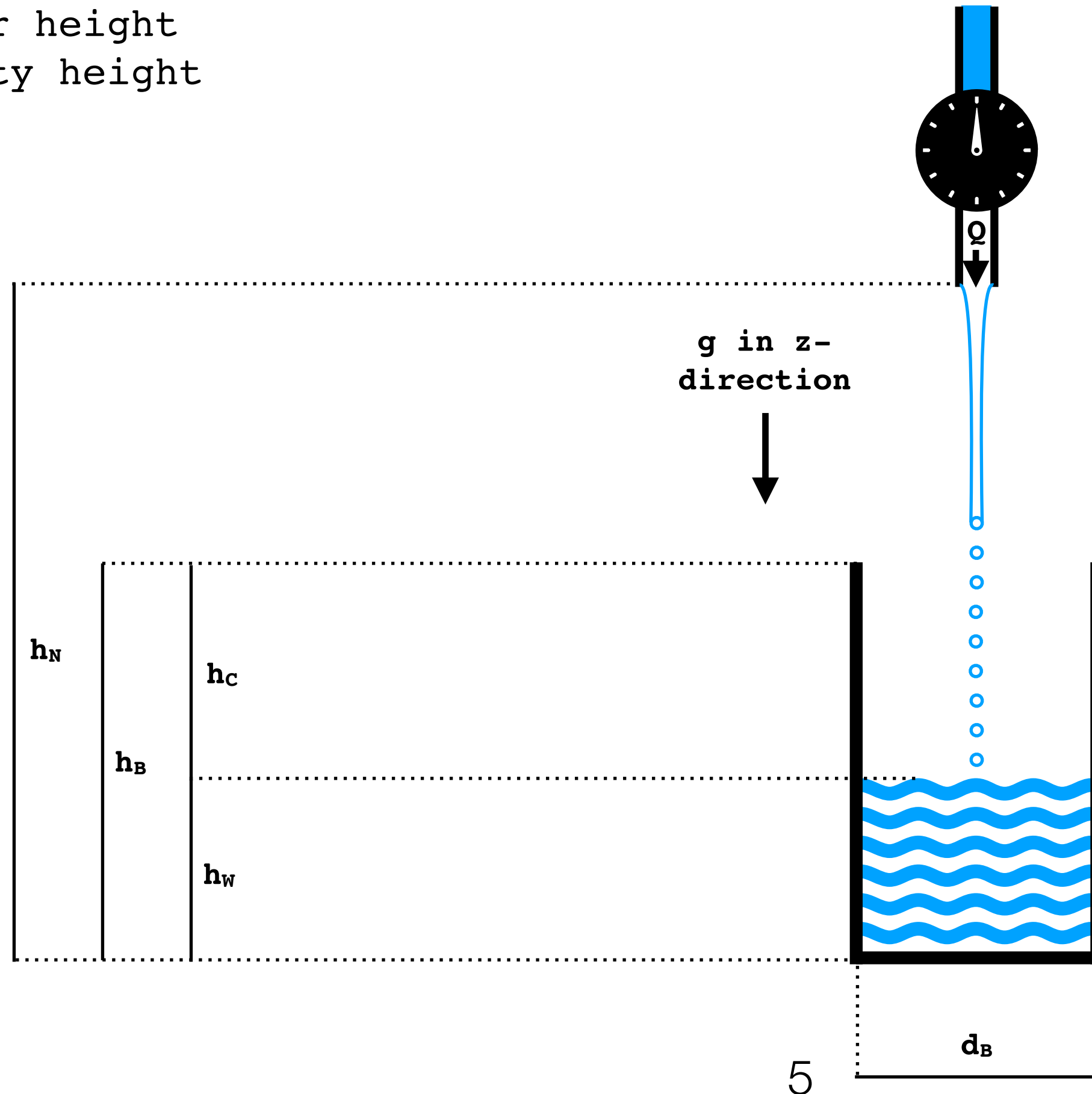


h_N : Nozzle height over container ground
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 d_B : Container-/ bottle-diameter
 h_W : Water height
 h_C : Cavity height

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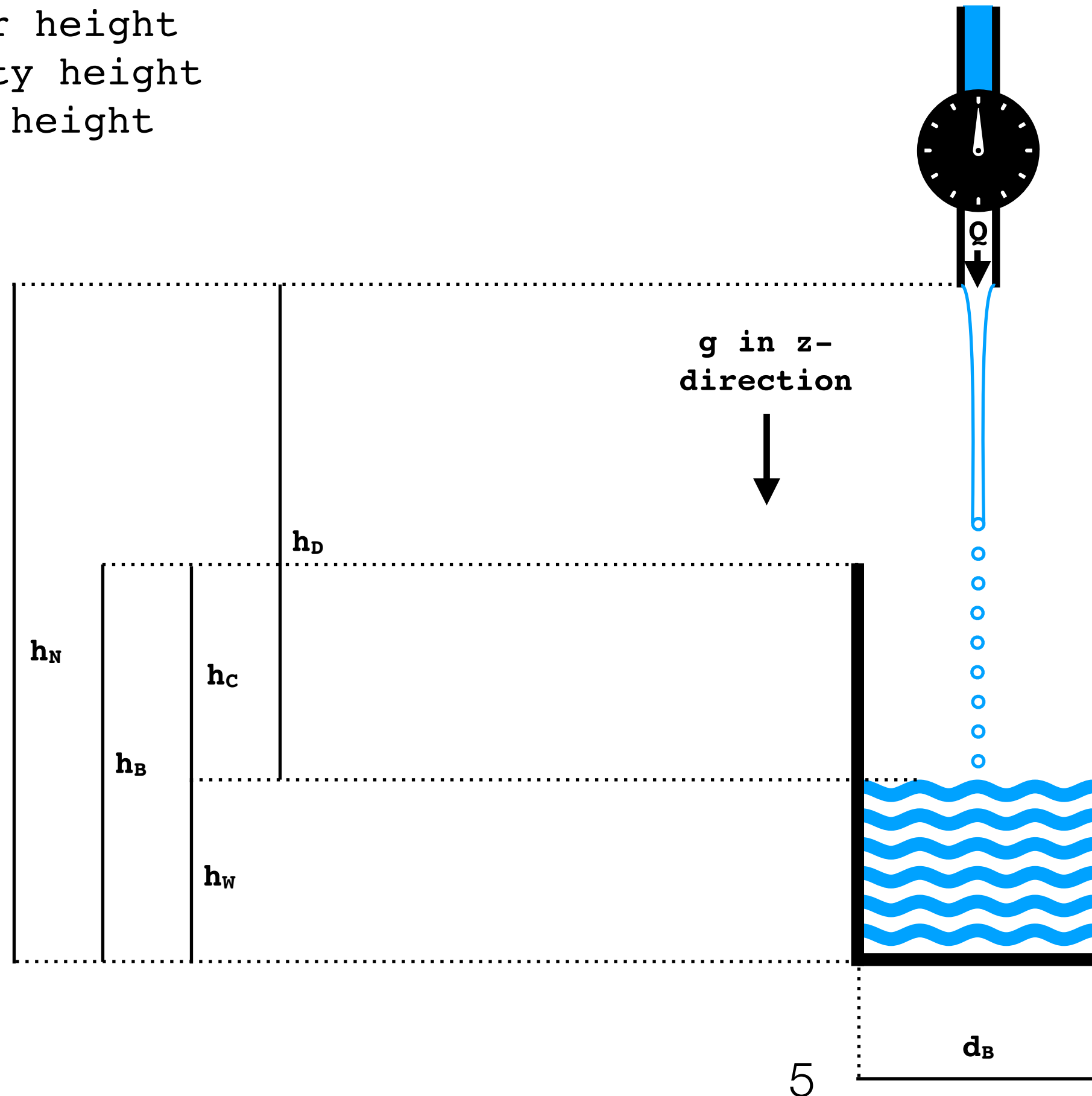


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 h_W : Water height
 h_C : Cavity height
 h_D : Drop height

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Standing waves

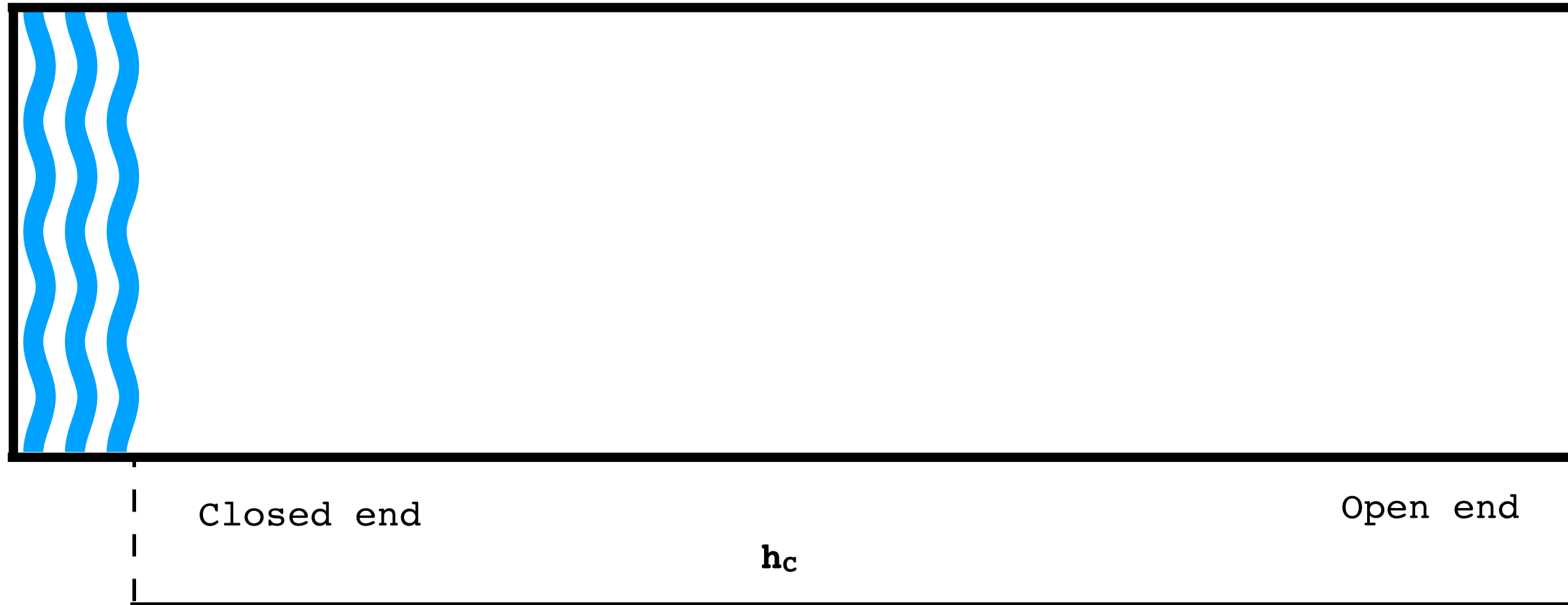


Standing waves



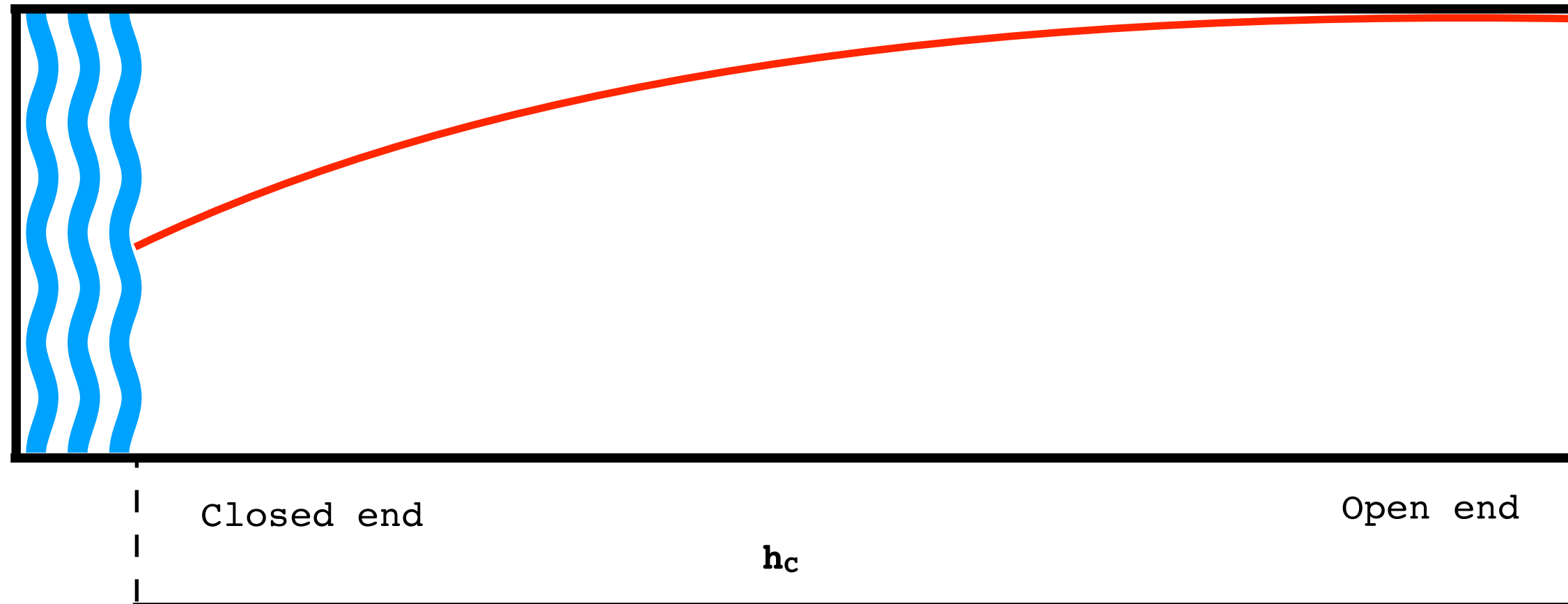


Standing waves



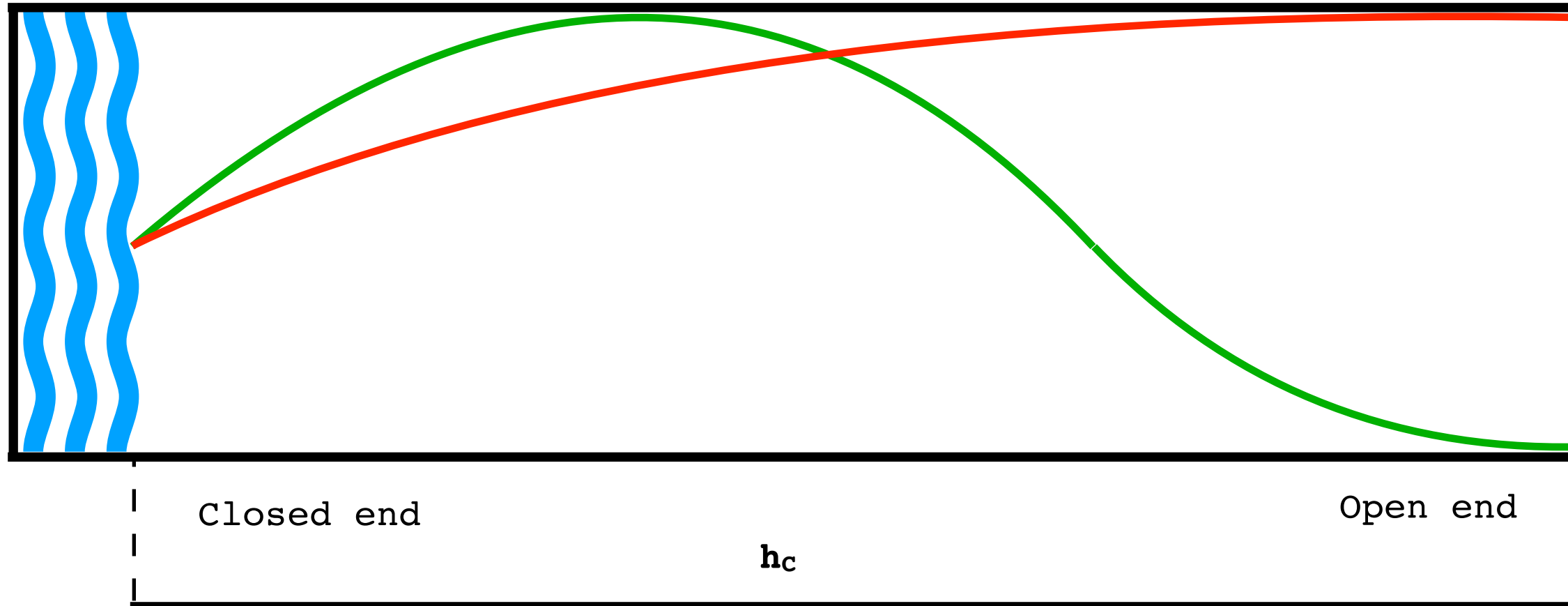


Standing waves



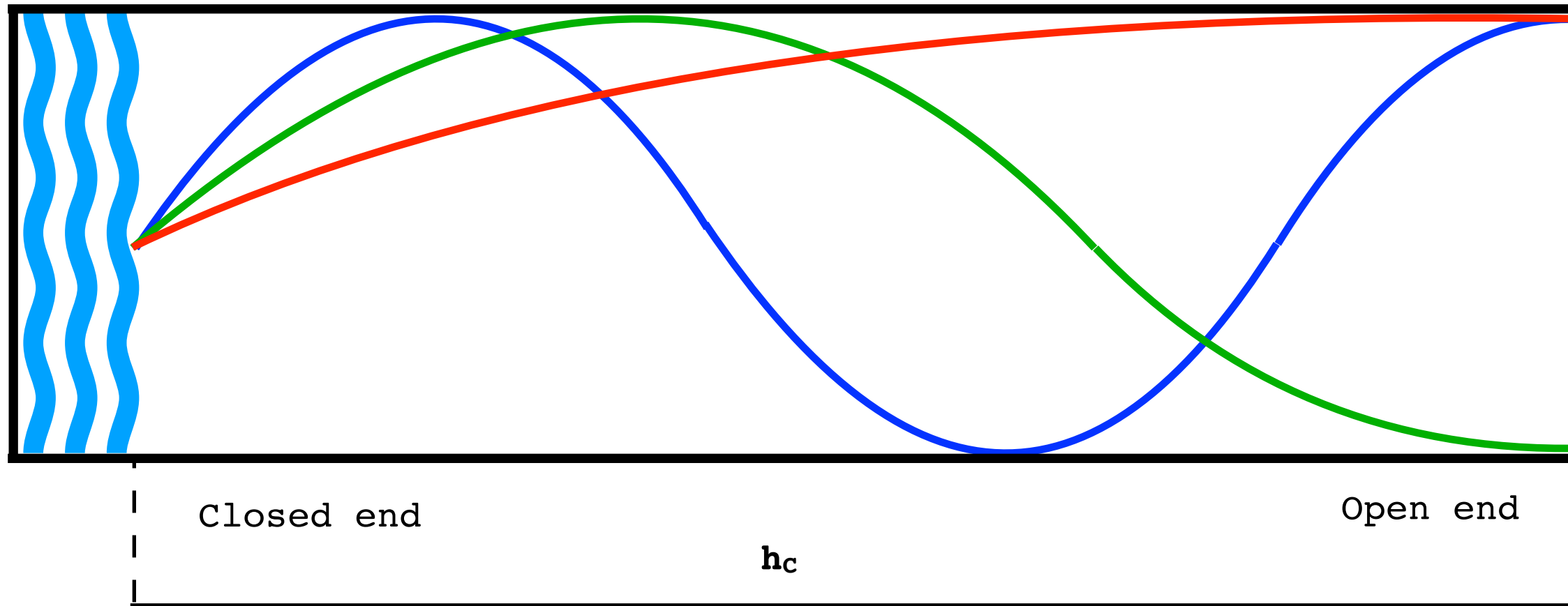


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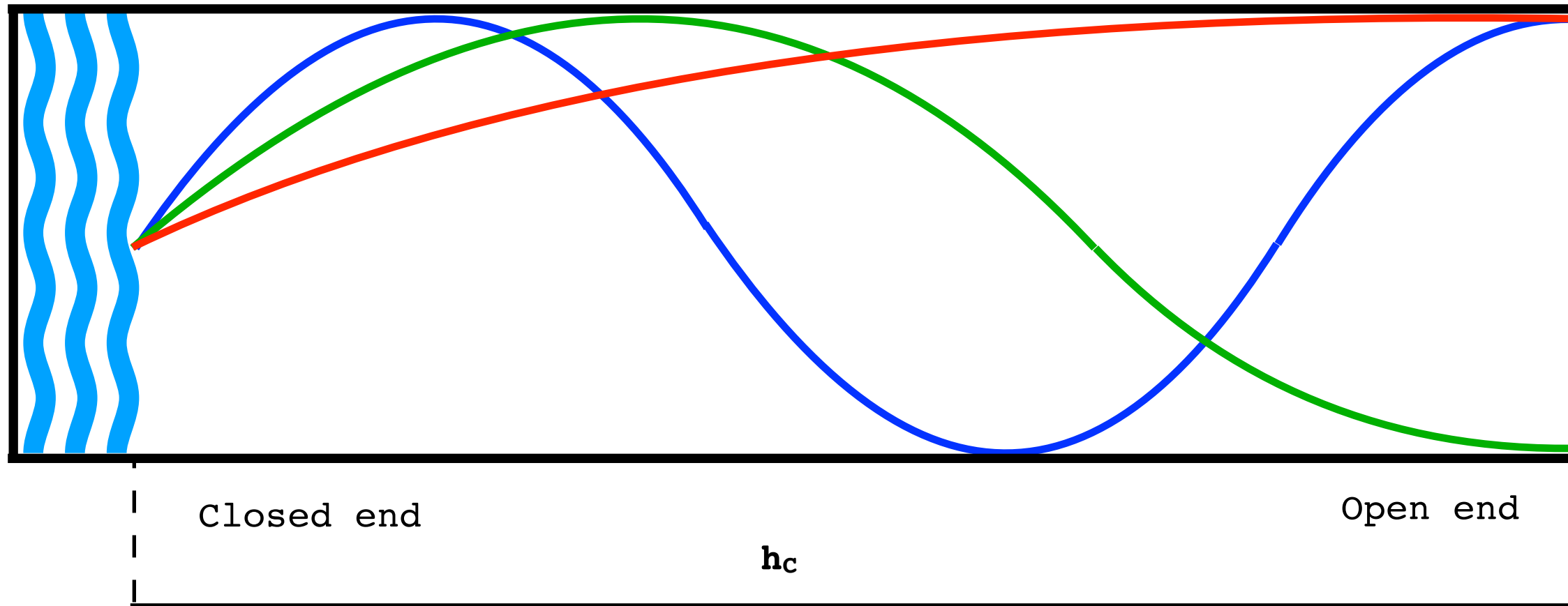


Standing waves





Standing waves



$$\lambda_0 = 4h_c$$

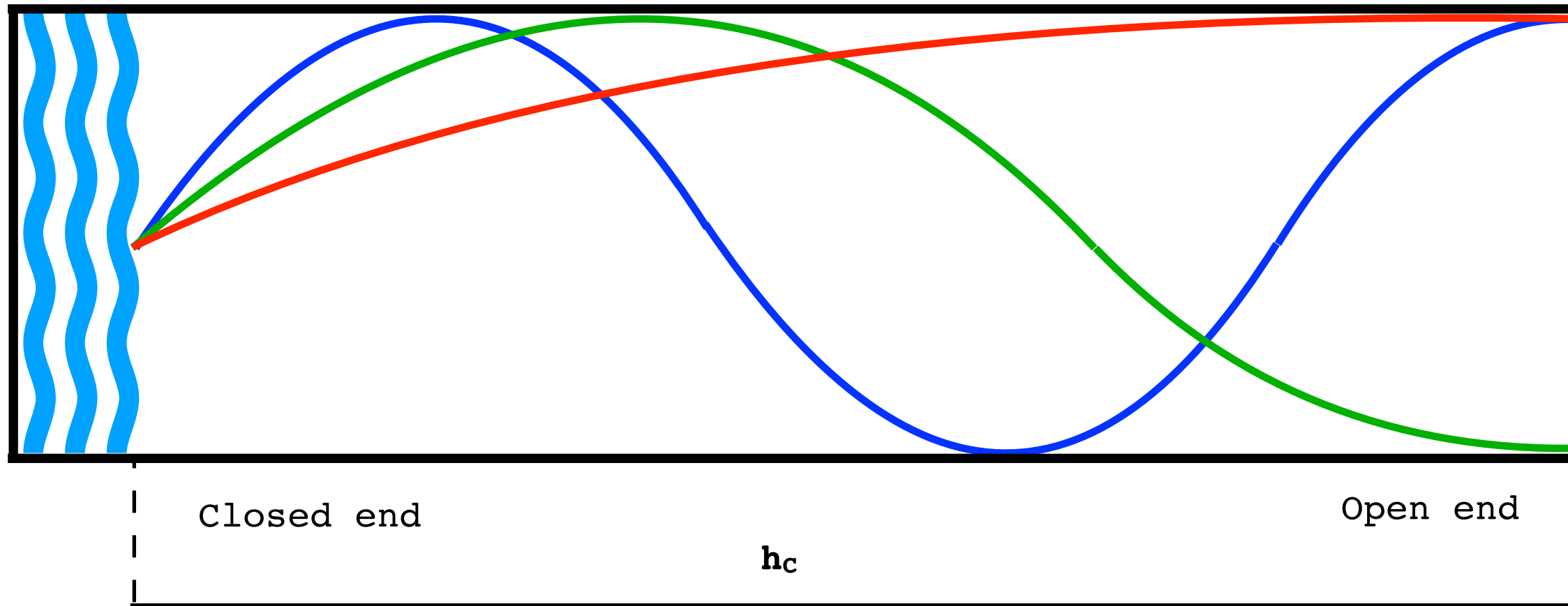
$$\lambda_1 = \frac{4}{3}h_c$$

$$\lambda_2 = \frac{4}{5}h_c$$

etc...



Standing waves



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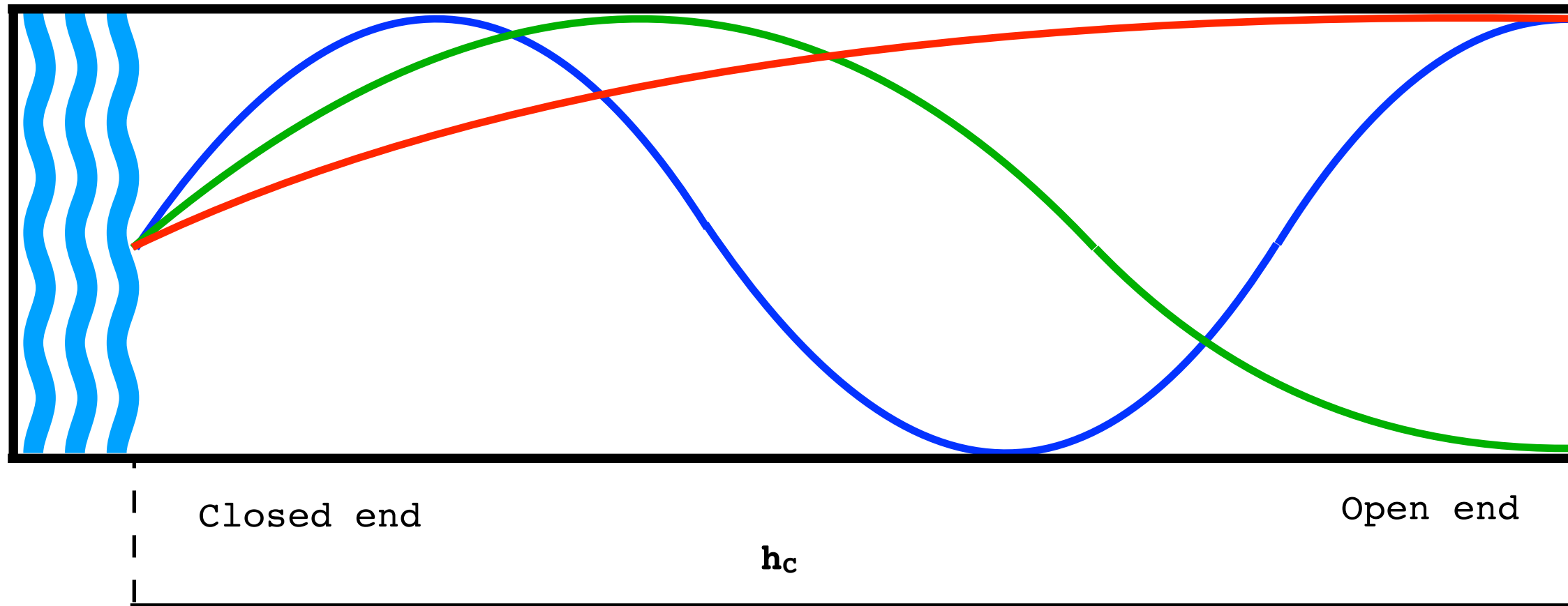
etc...

$$f = \frac{c}{\lambda}$$

→



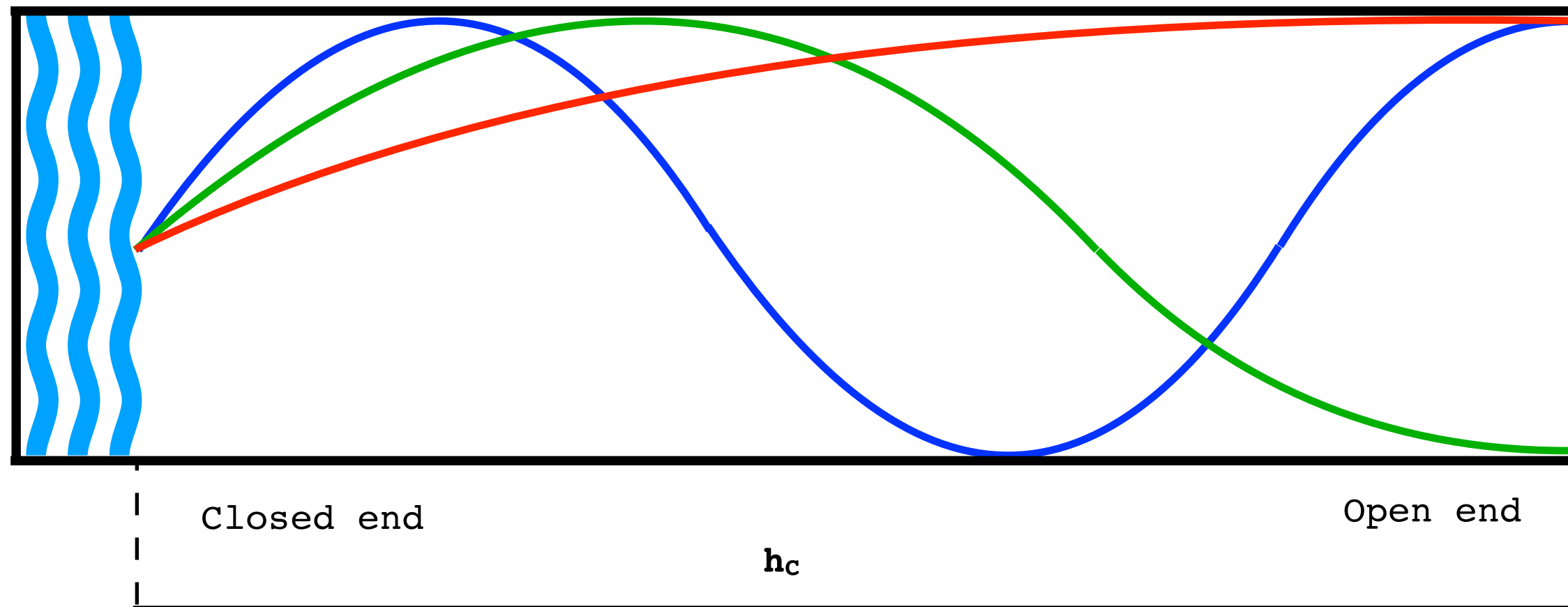
Standing waves



$$\begin{array}{l} \lambda_0 = 4h_c \\ \lambda_1 = \frac{4}{3}h_c \\ \lambda_2 = \frac{4}{5}h_c \\ \text{etc...} \end{array} \xrightarrow{f = \frac{c}{\lambda}} \begin{array}{l} f_0 = \frac{c}{4h_c} \\ f_1 = \frac{3c}{4h_c} = 3f_0 \\ f_2 = \frac{5c}{4h_c} = 5f_0 \end{array}$$



Standing waves



$$\lambda_0 = 4h_c$$

$$\lambda_1 = \frac{4}{3}h_c$$

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etc...

$$f = \frac{c}{\lambda}$$

$$f_0 = \frac{c}{4h_c}$$

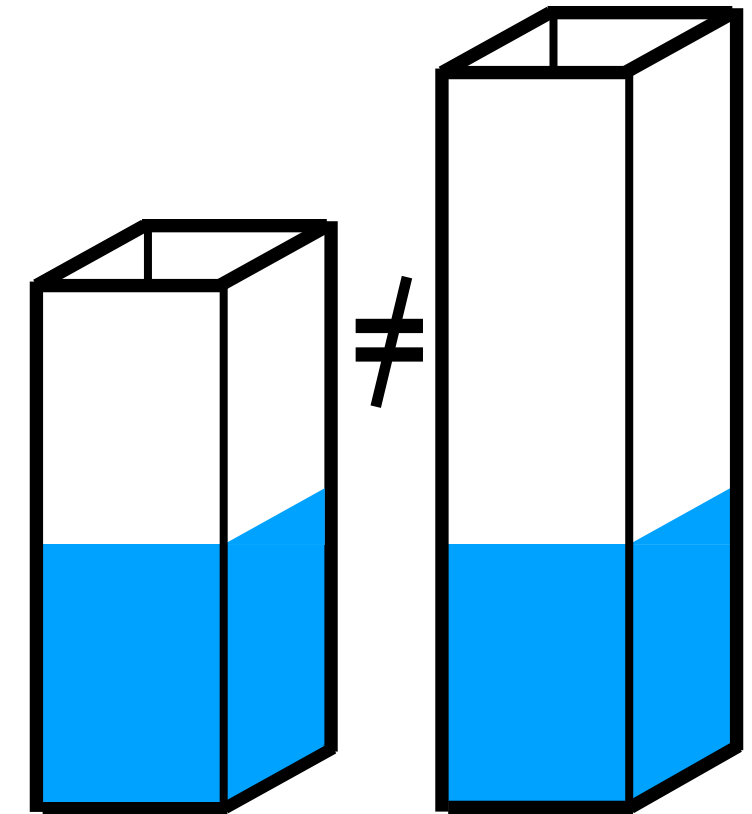
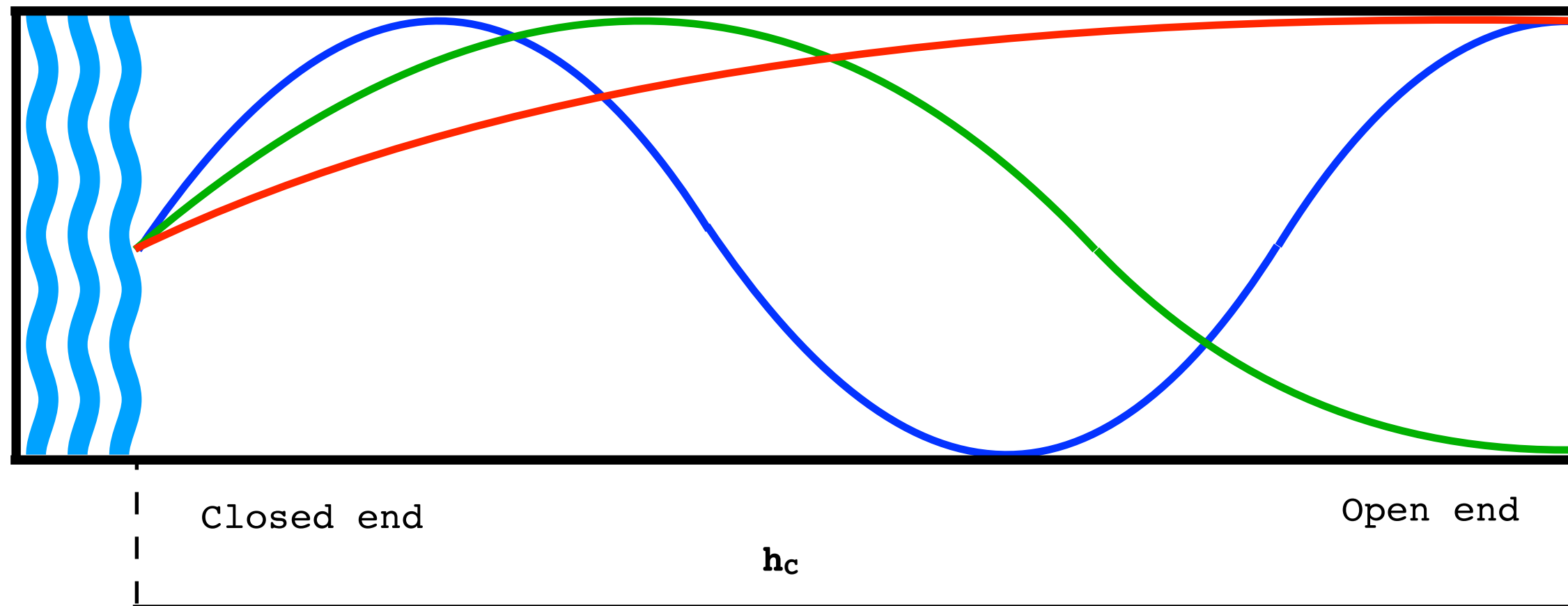
$$f_1 = \frac{3c}{4h_c} = 3f_0$$

$$f_2 = \frac{5c}{4h_c} = 5f_0$$

Standing waves in the cavity change their frequencies based on the changing water height.



Standing waves



$$\lambda_0 = 4h_c$$

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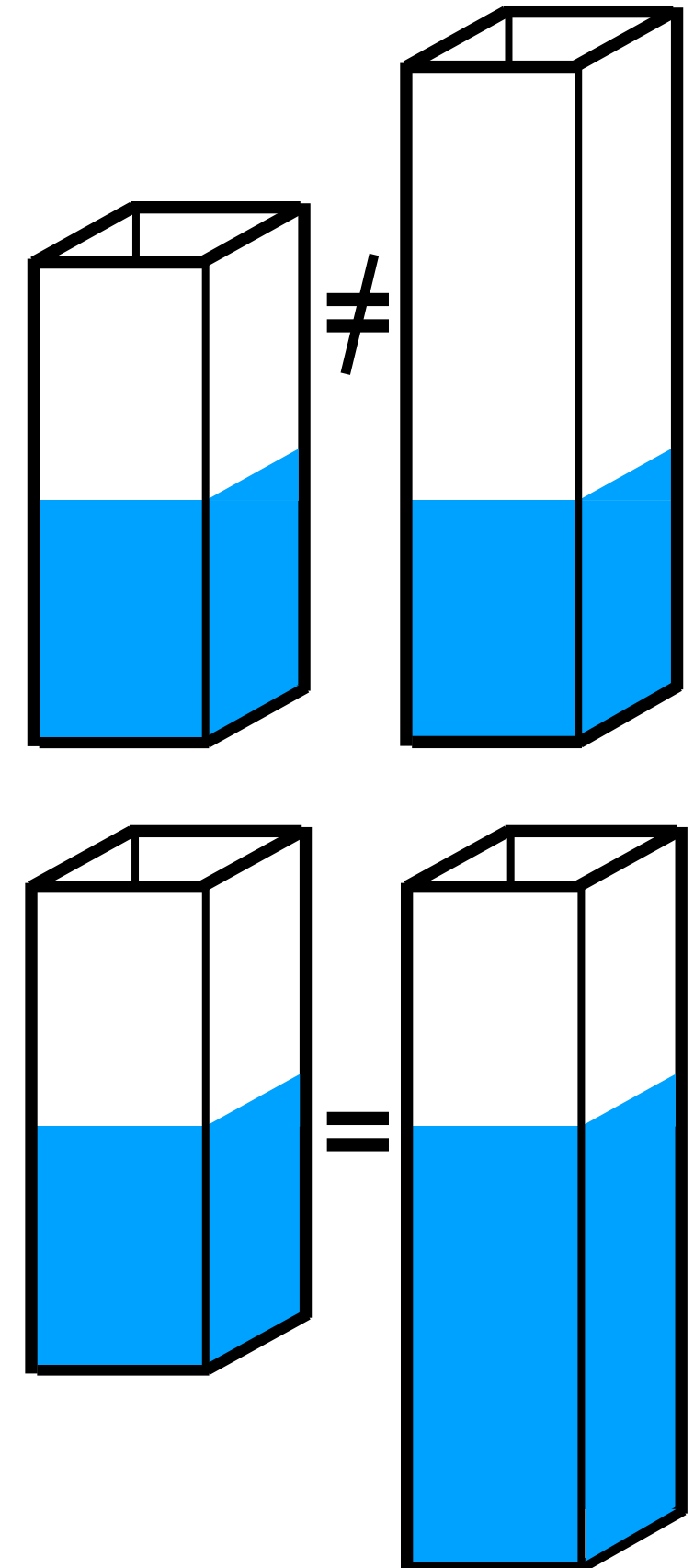
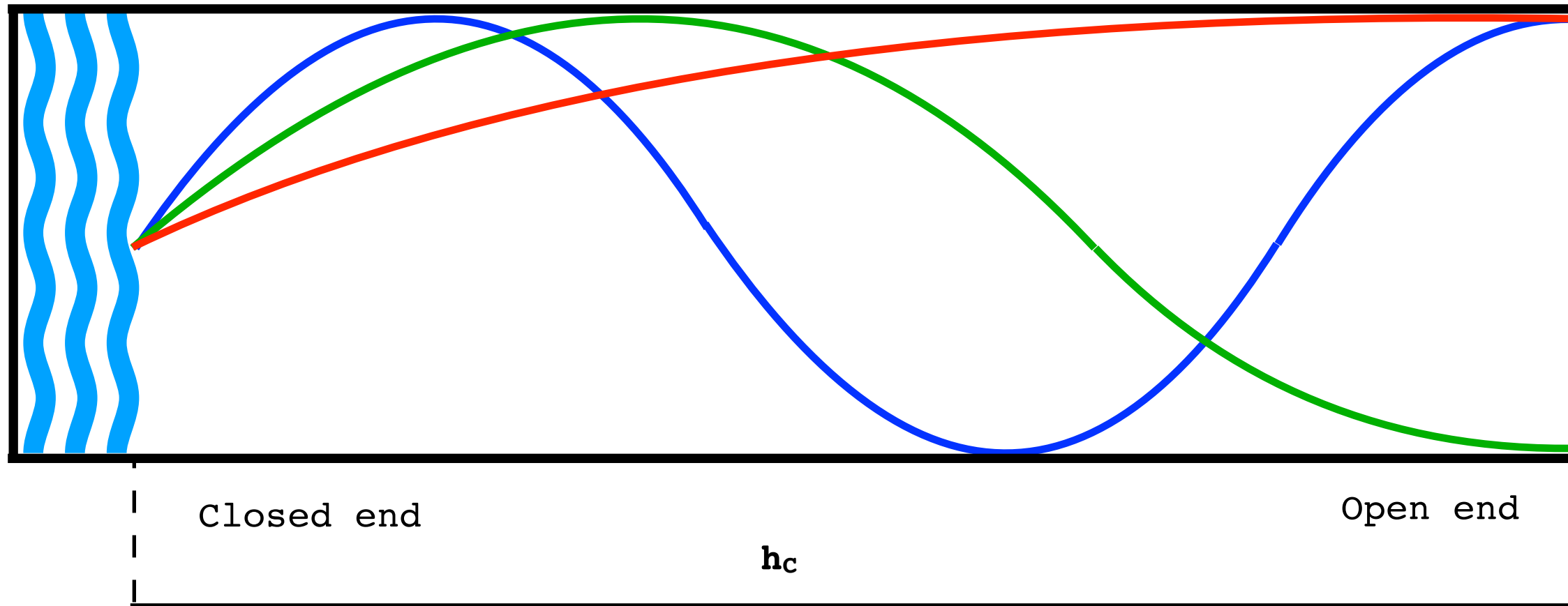
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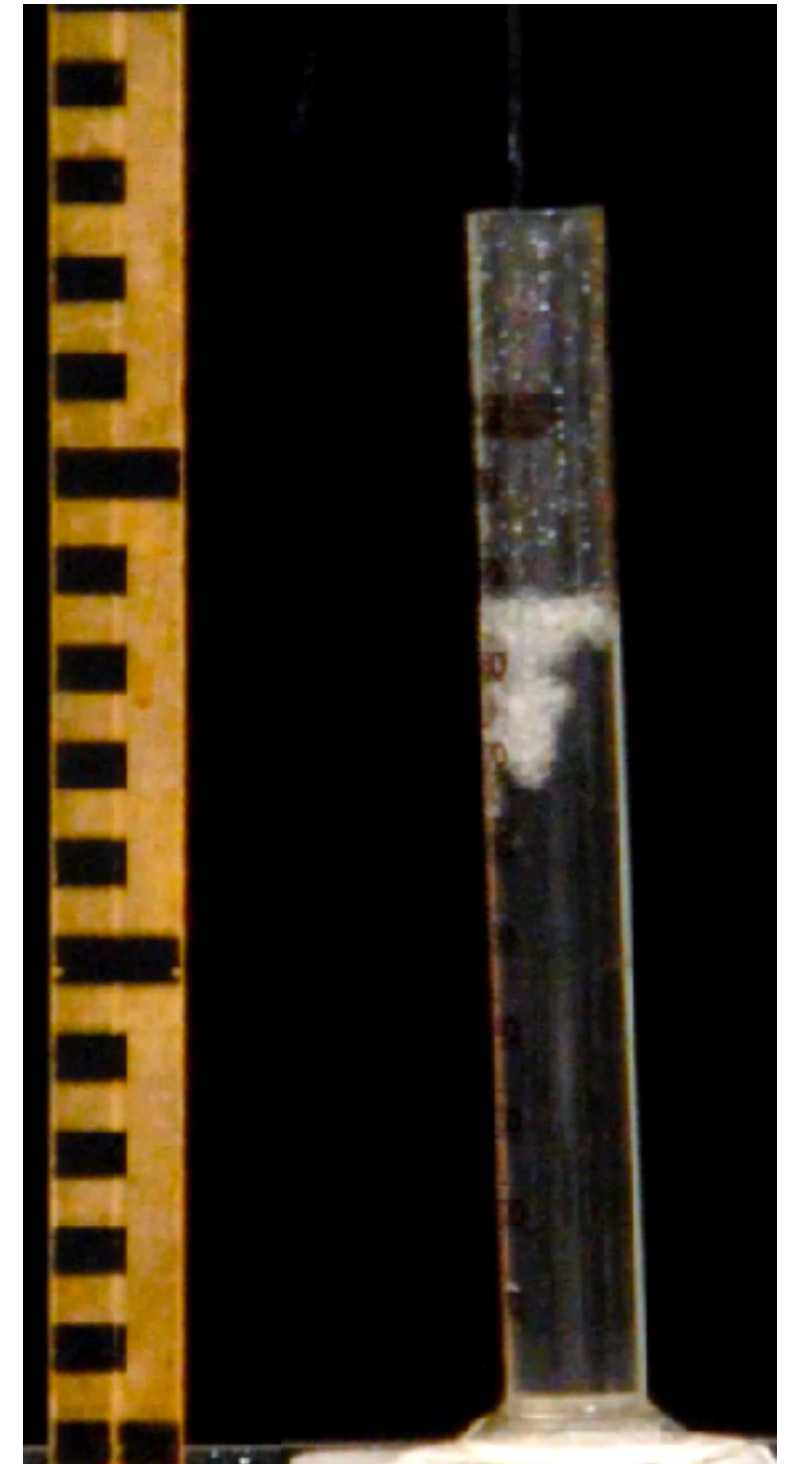
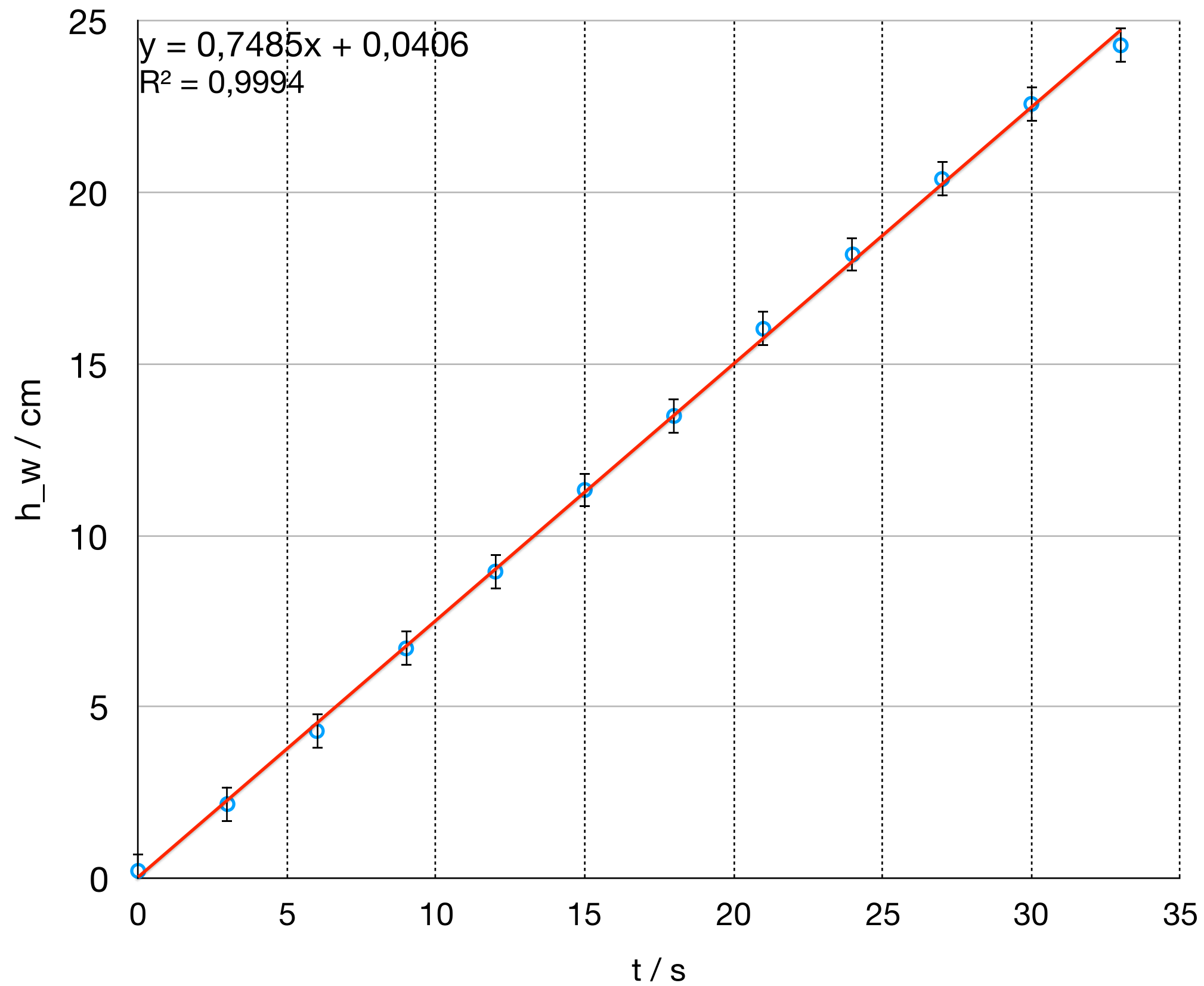
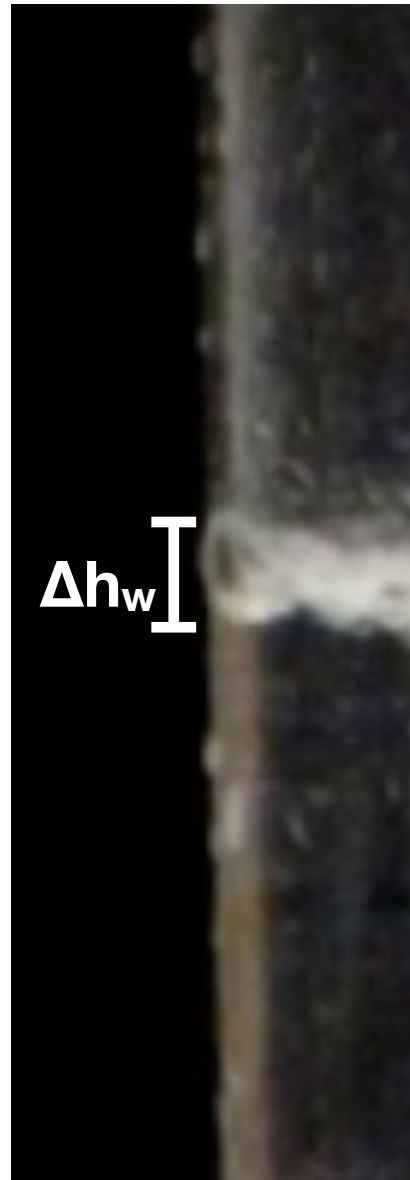


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Standing waves in the cavity change their frequencies based on the changing water height.

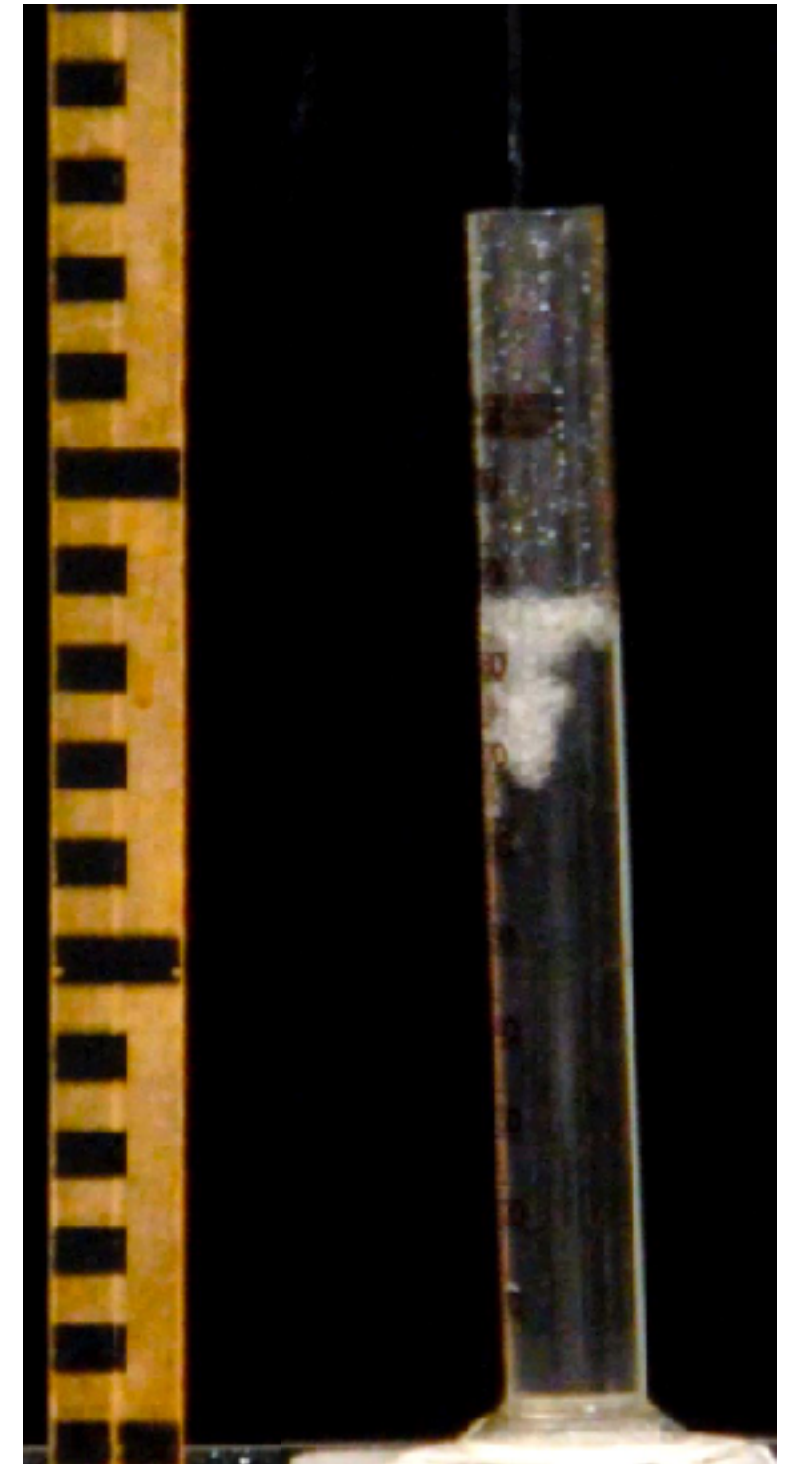
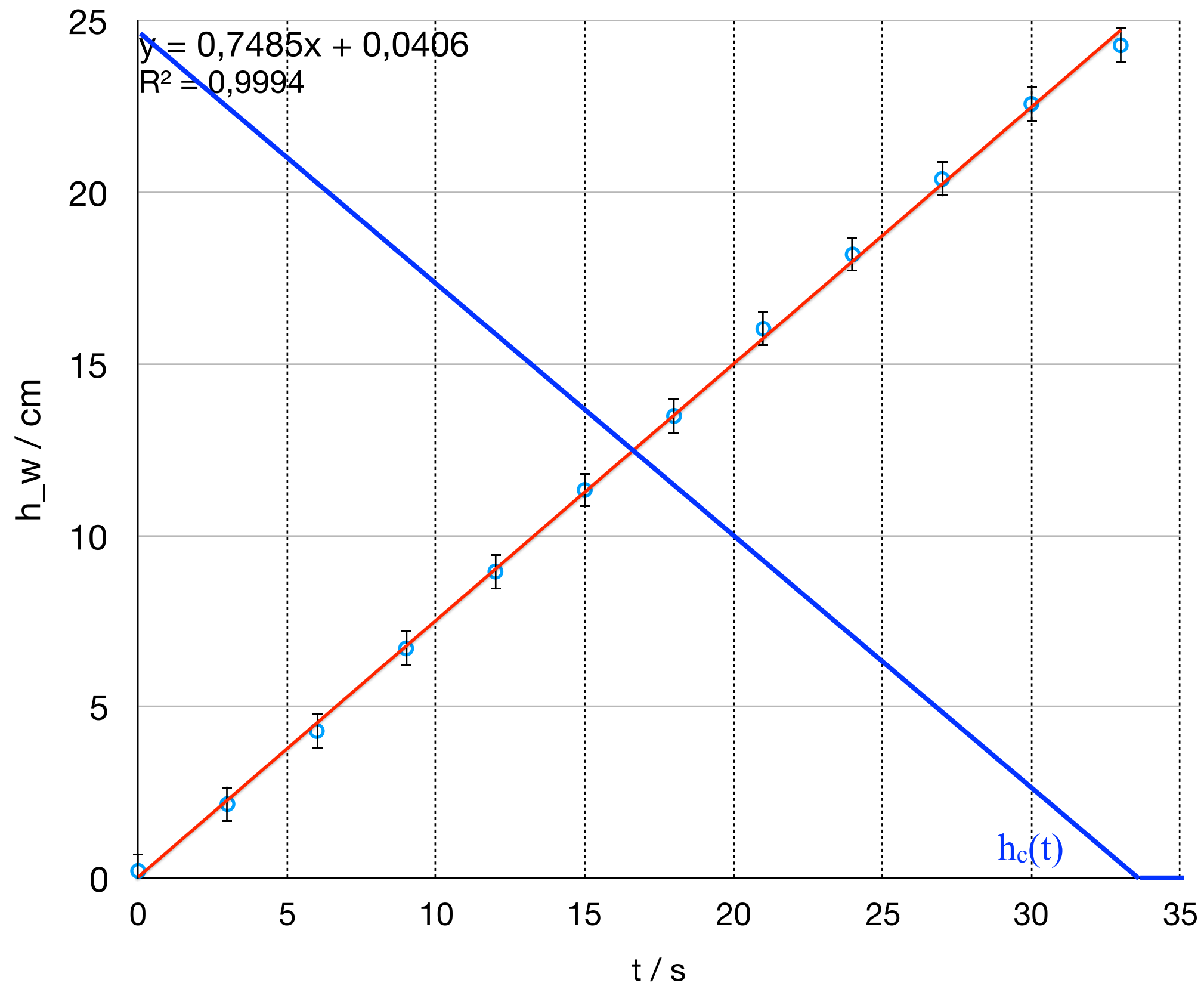
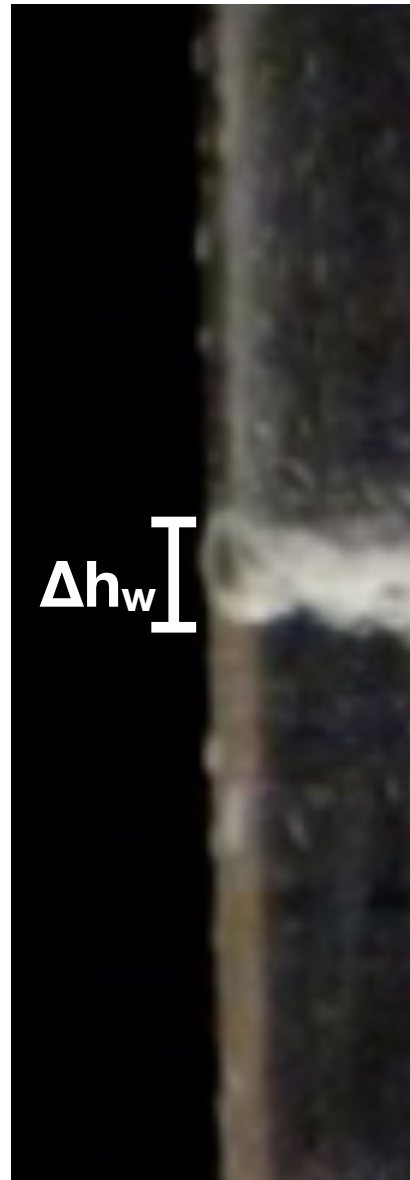


Water height over time





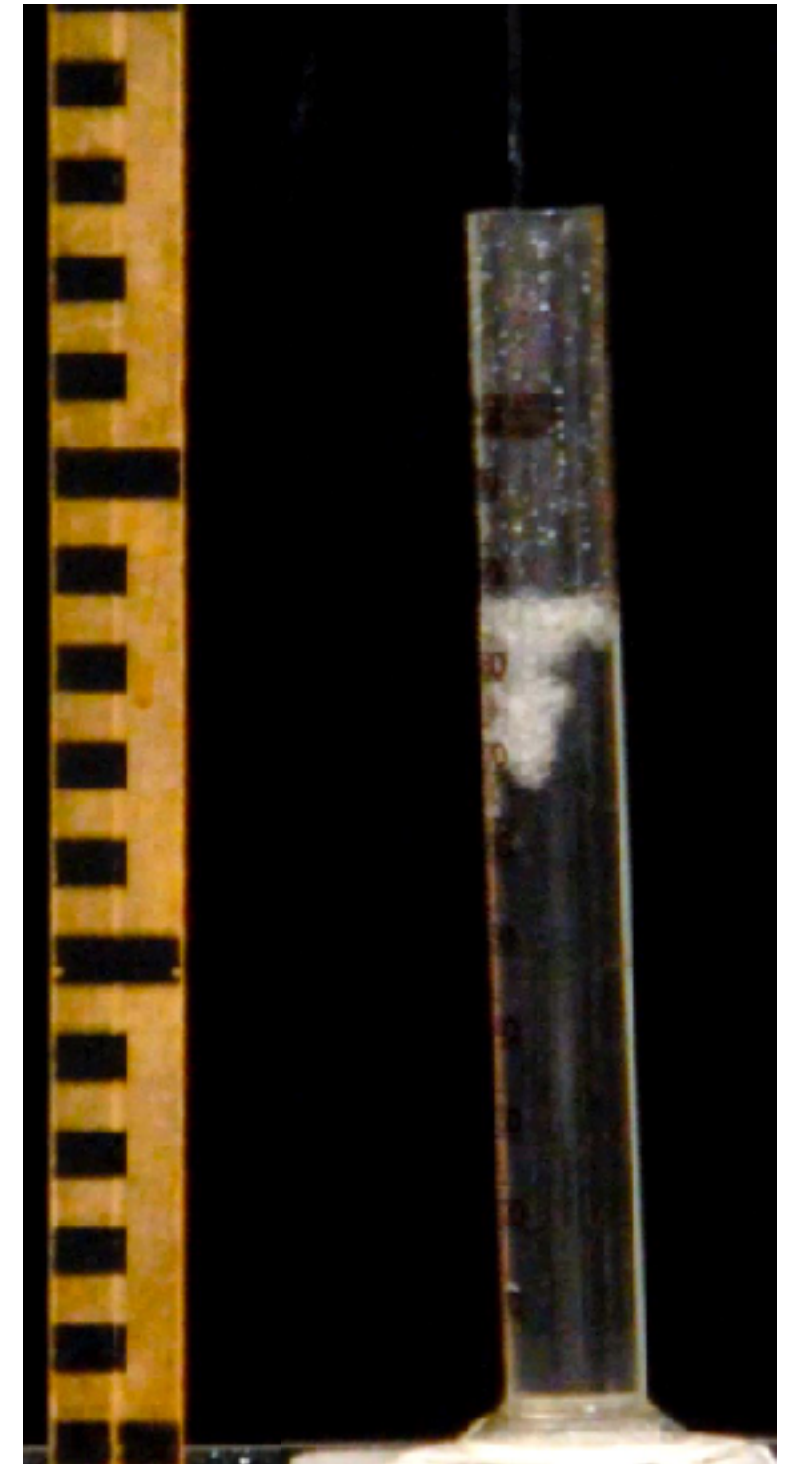
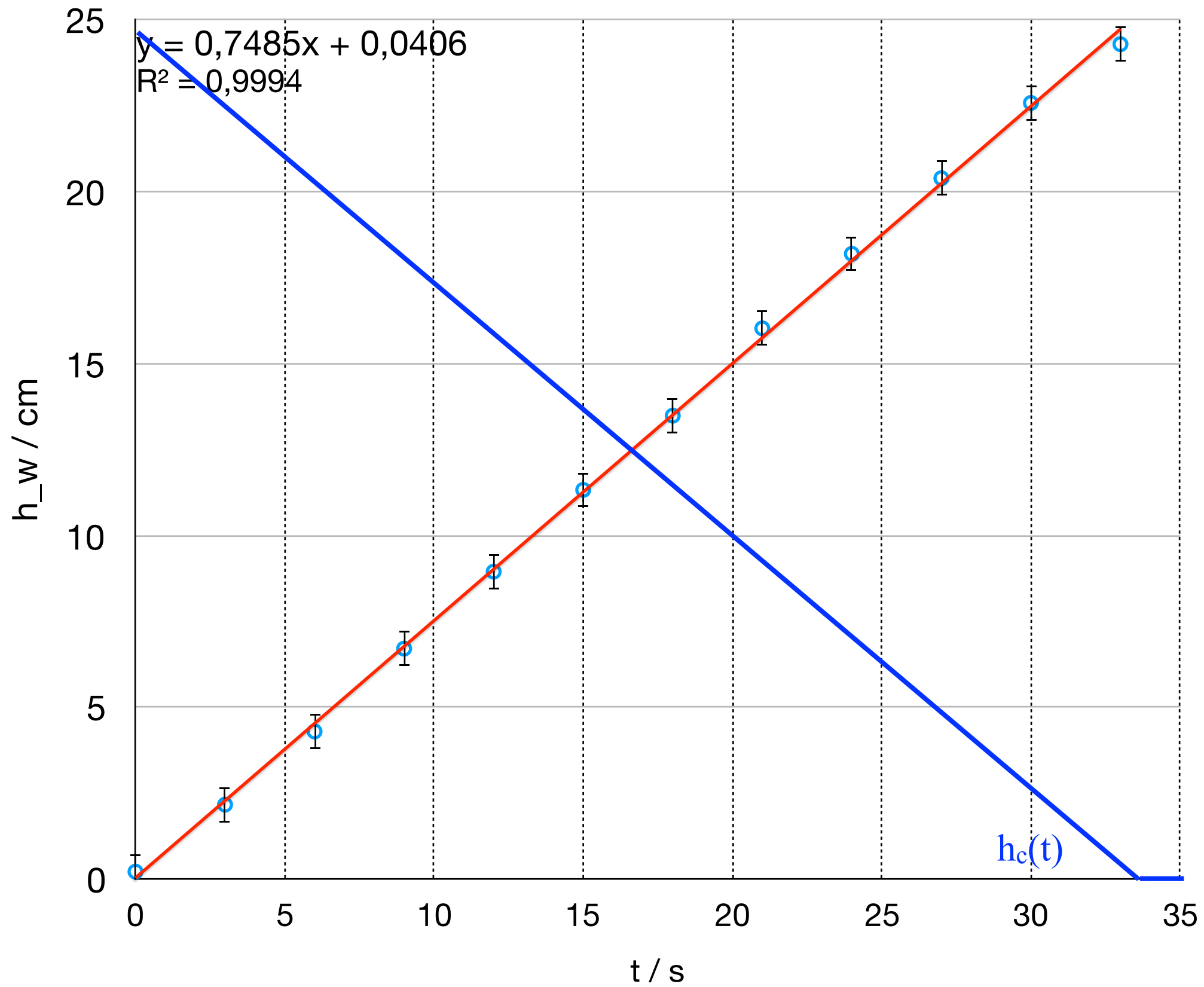
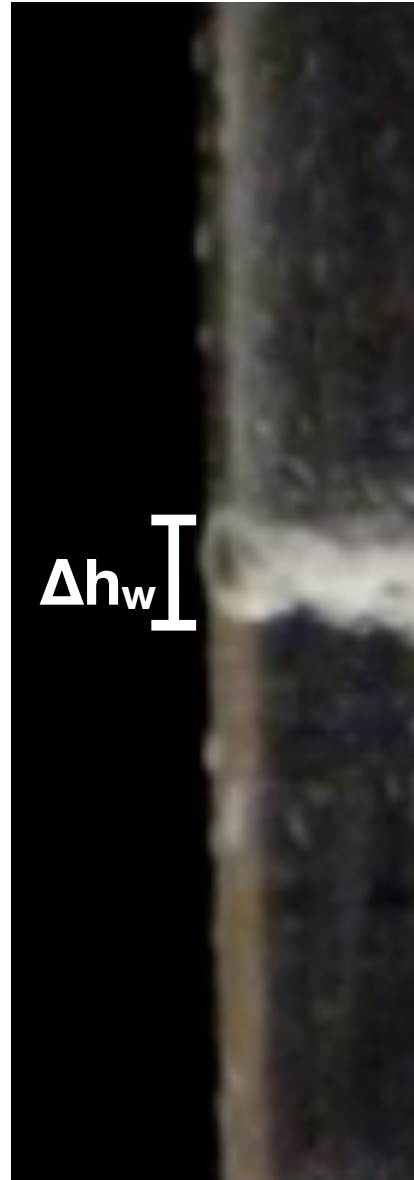
Water height over time

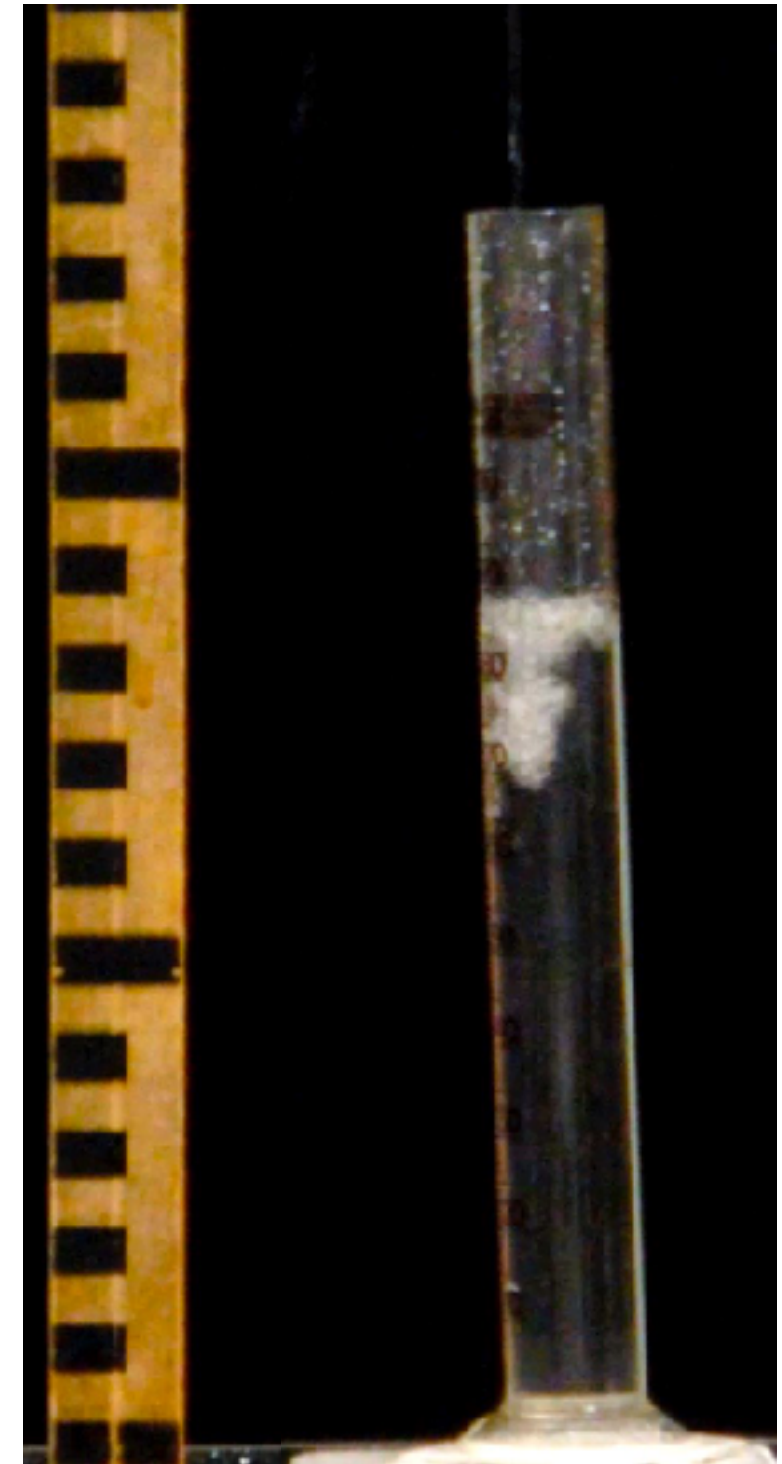
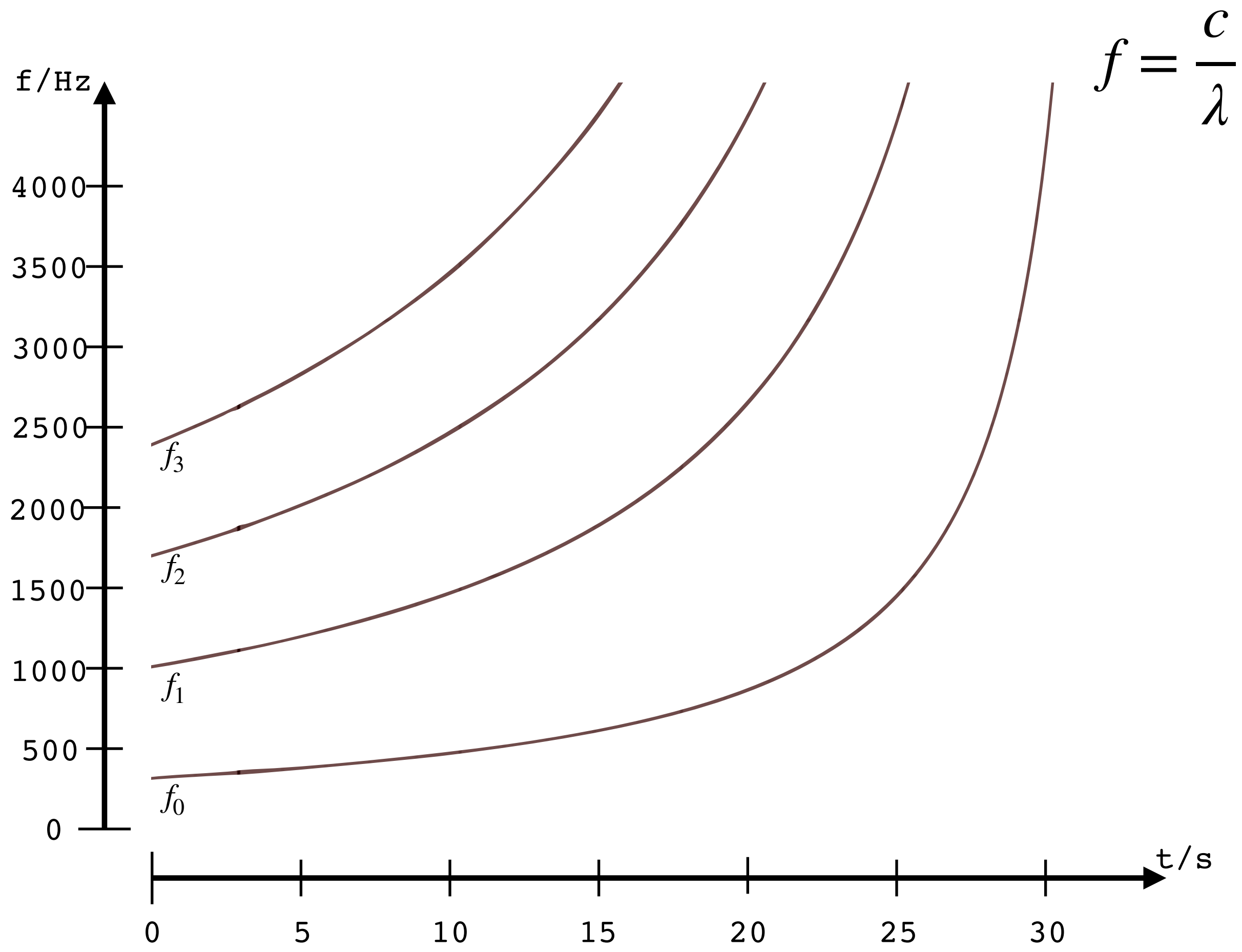




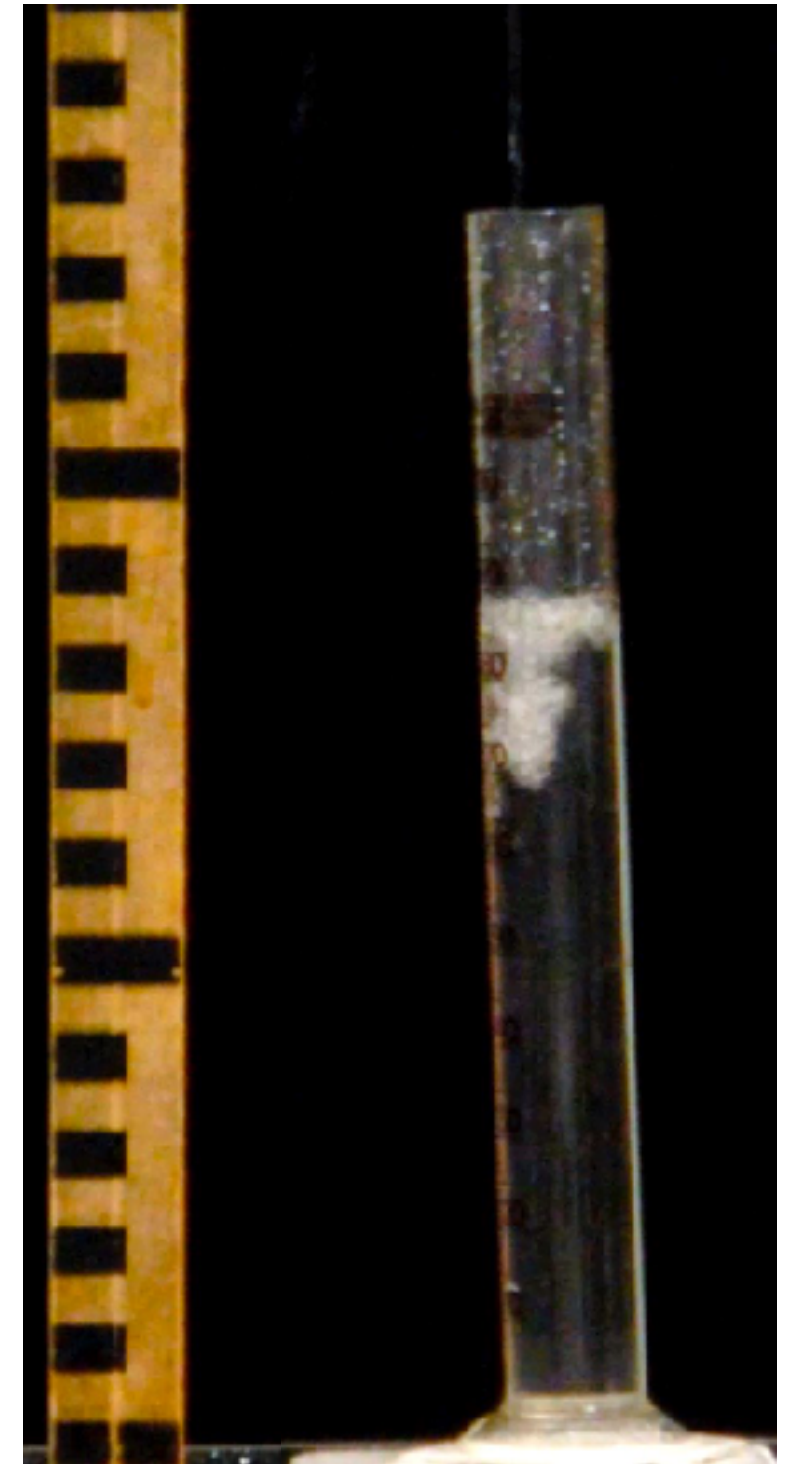
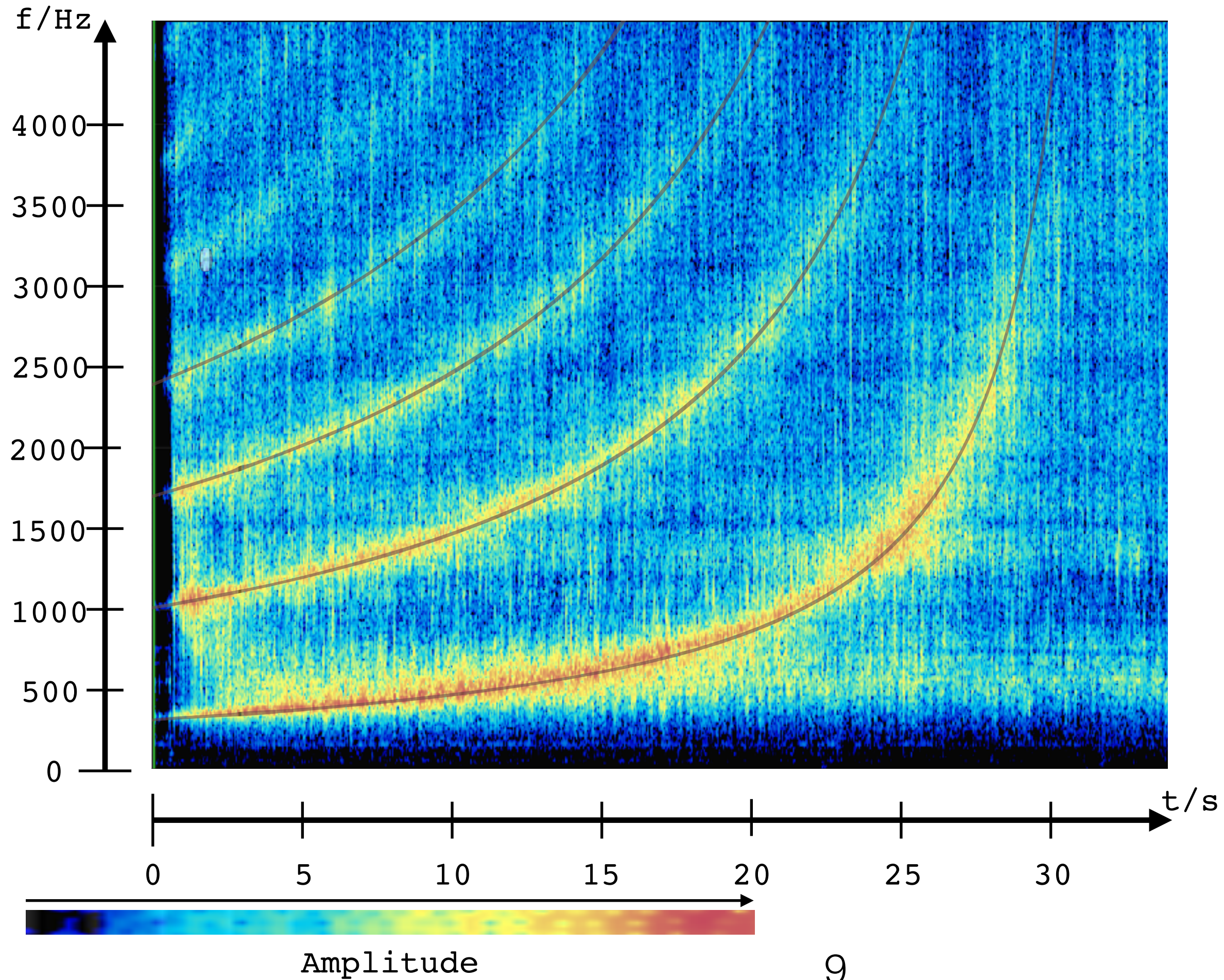
Water height over time

$$f = \frac{c}{\lambda}$$

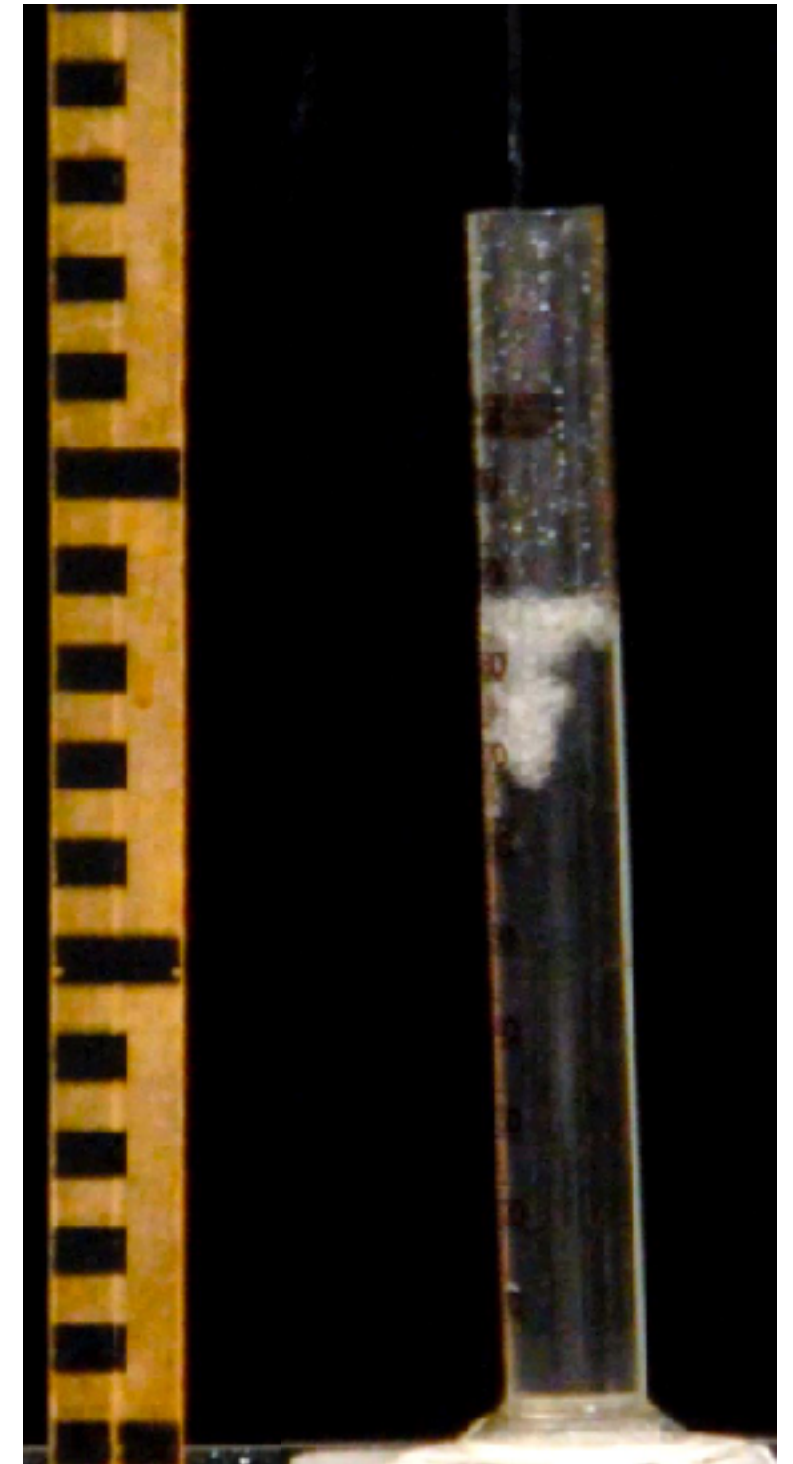
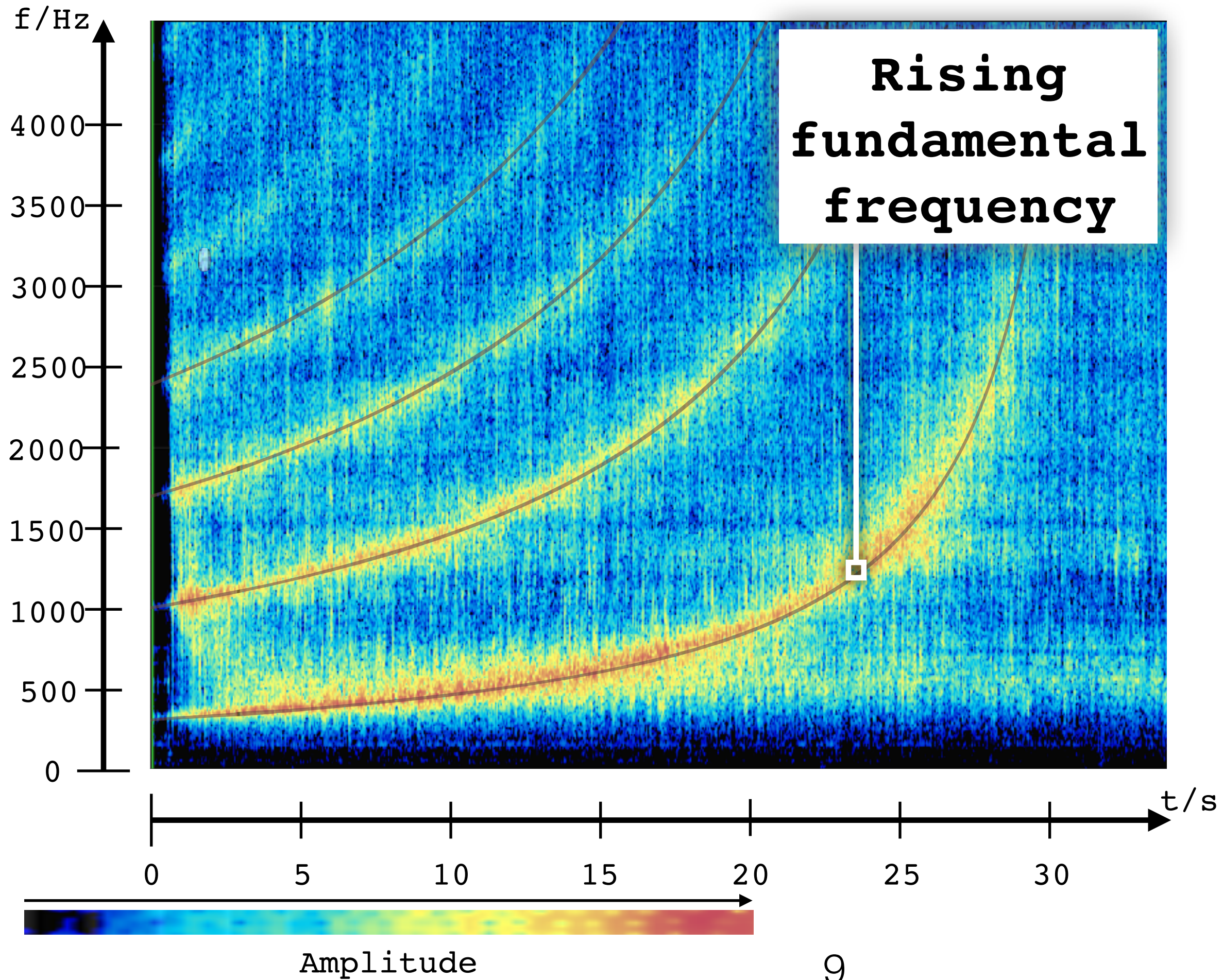




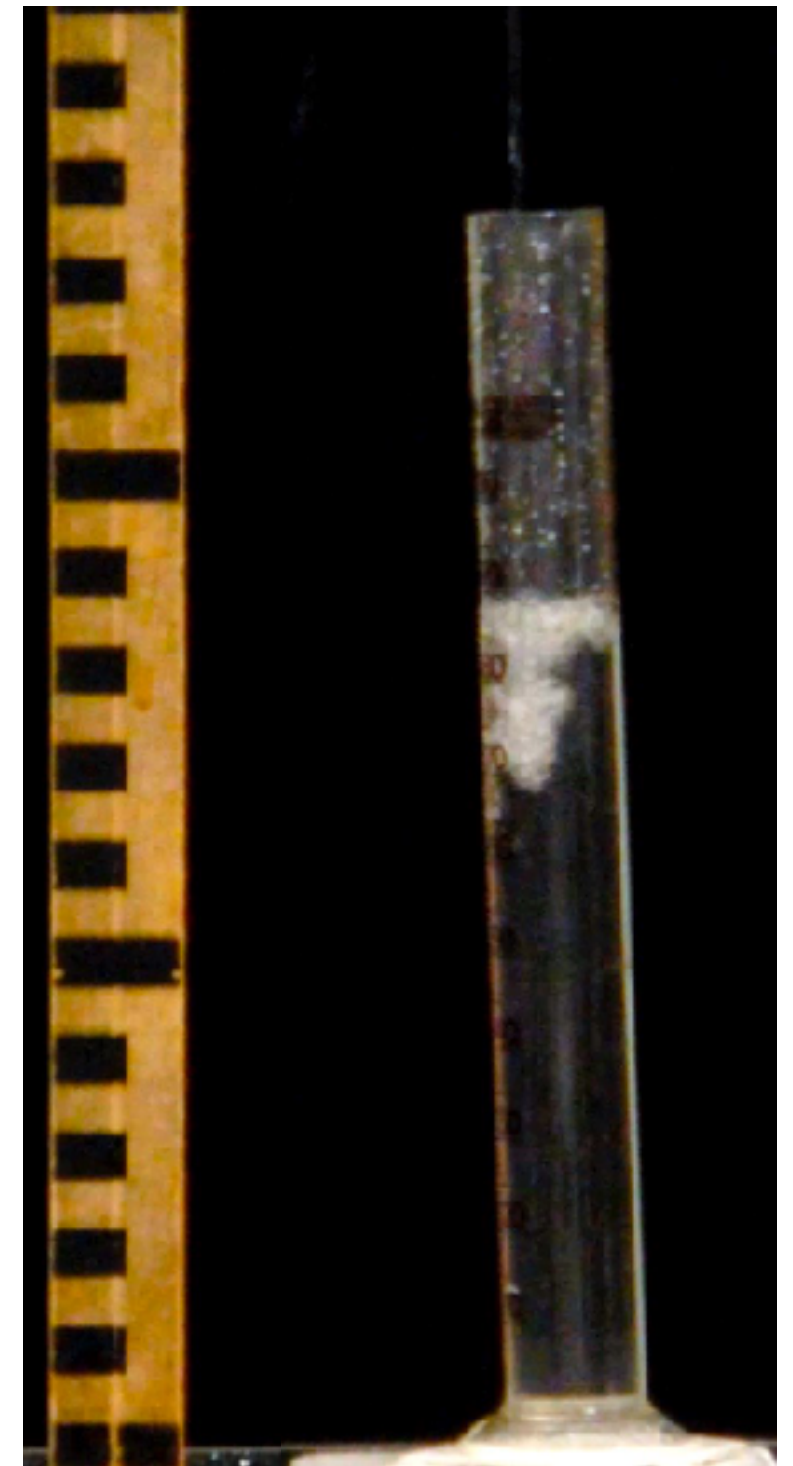
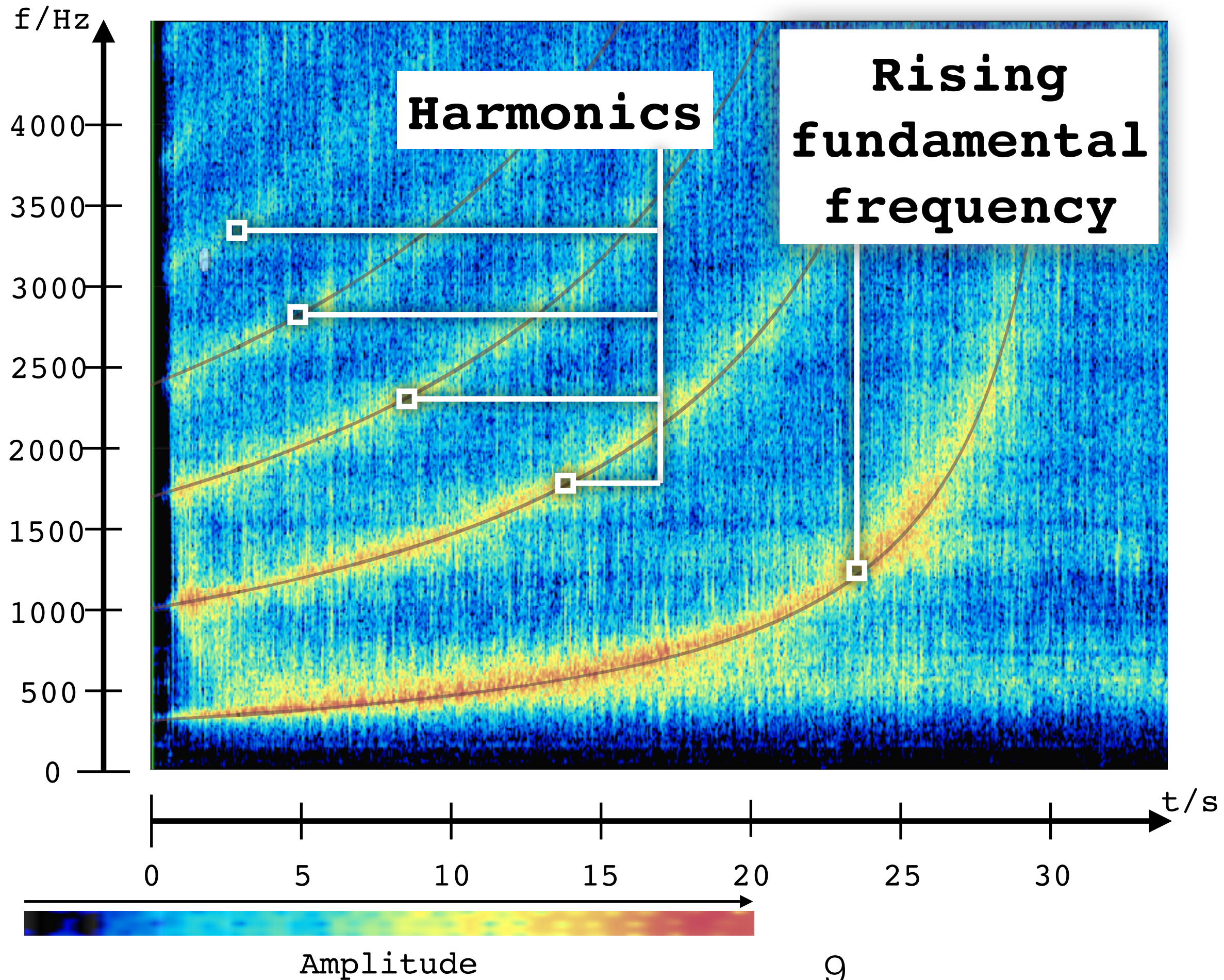
$$Q = 1.04 \frac{l}{min} \pm 0.04 \frac{l}{min}$$



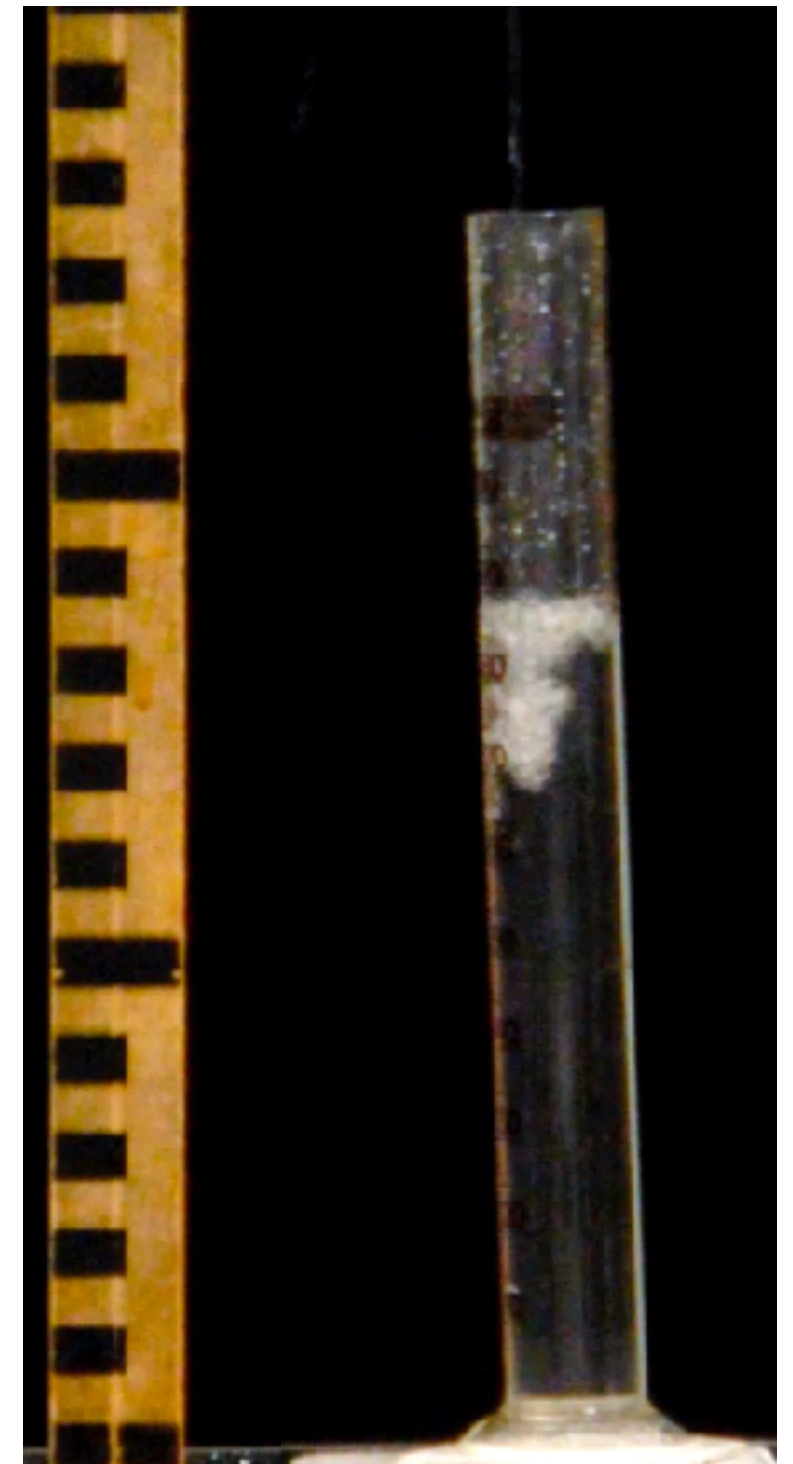
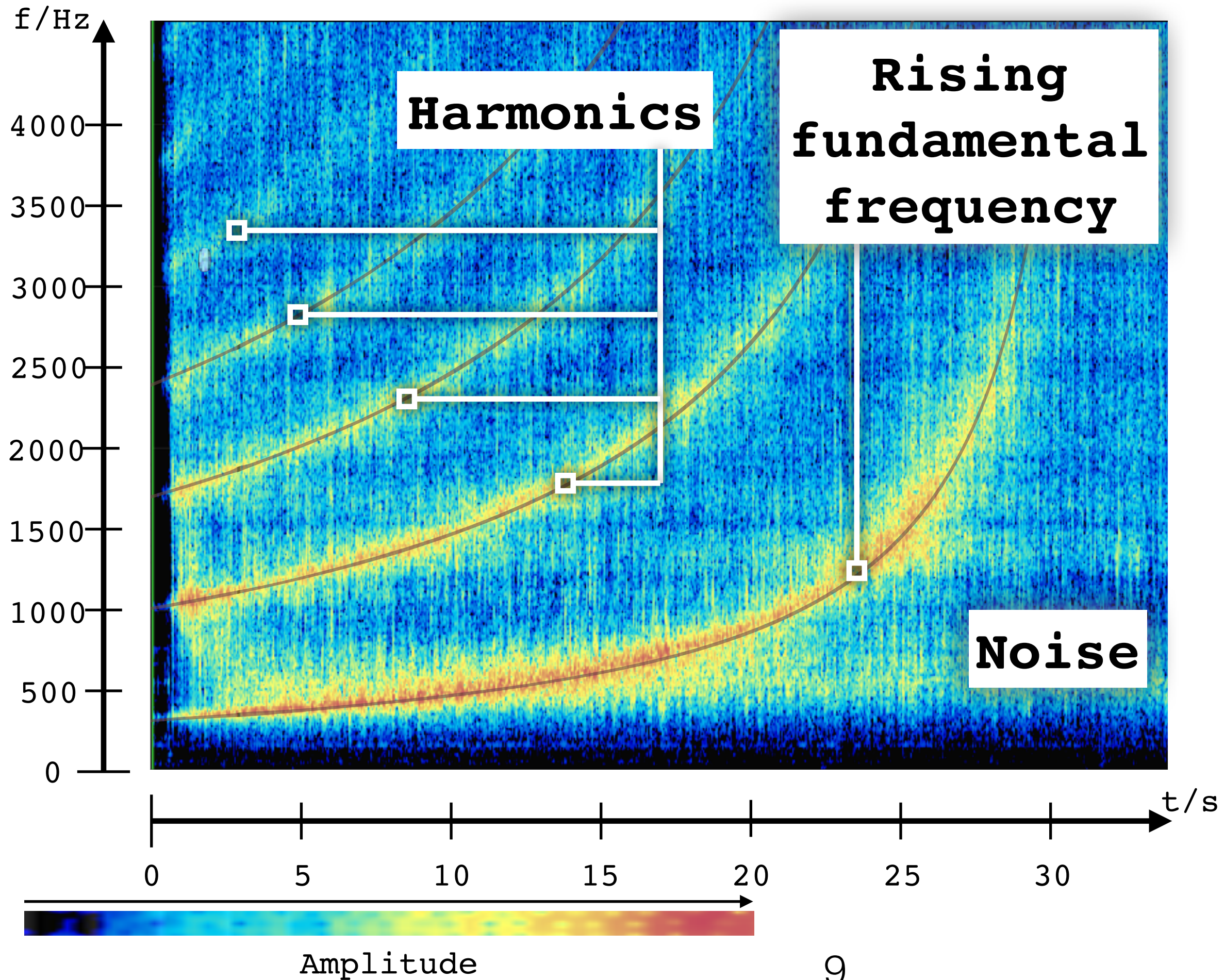
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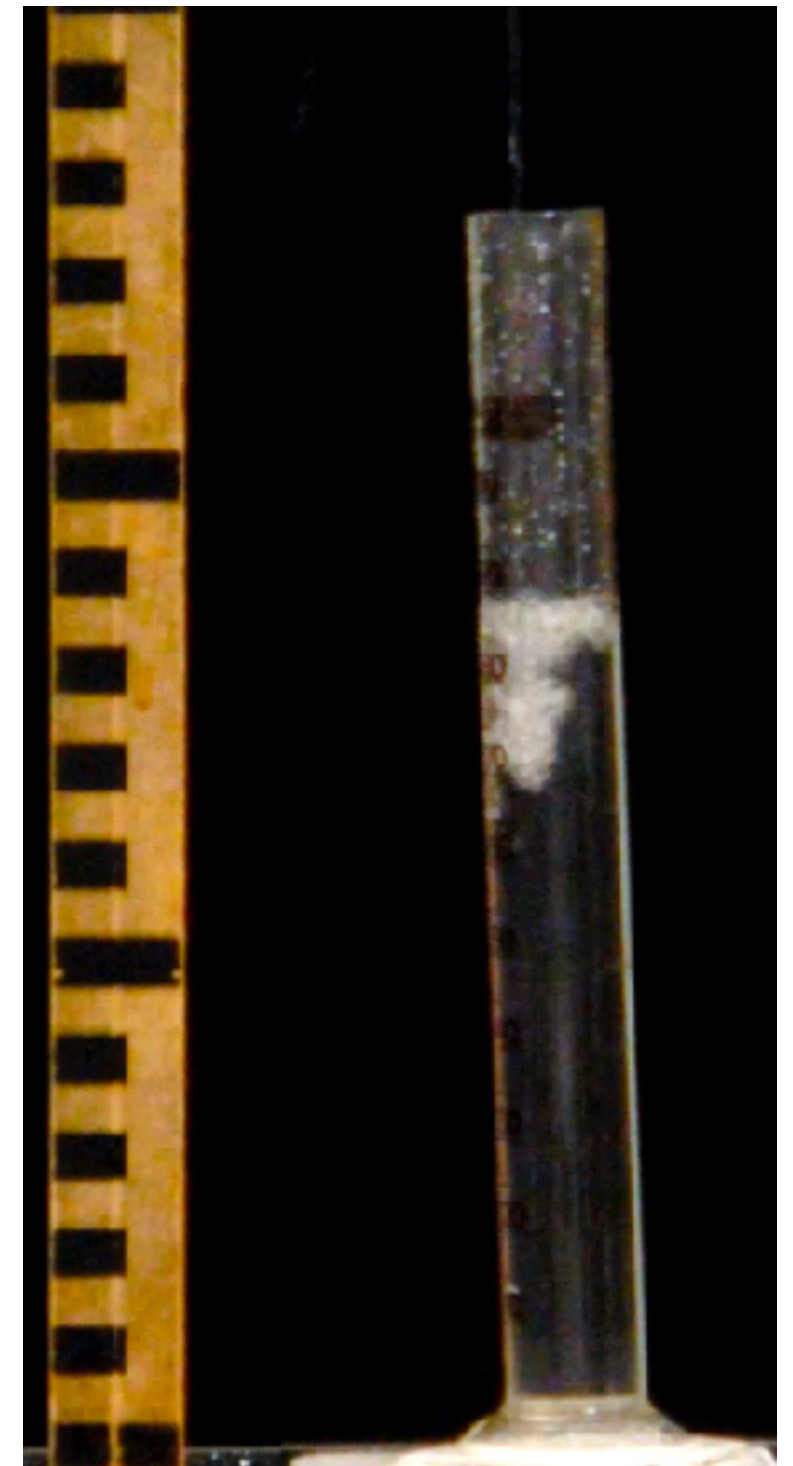
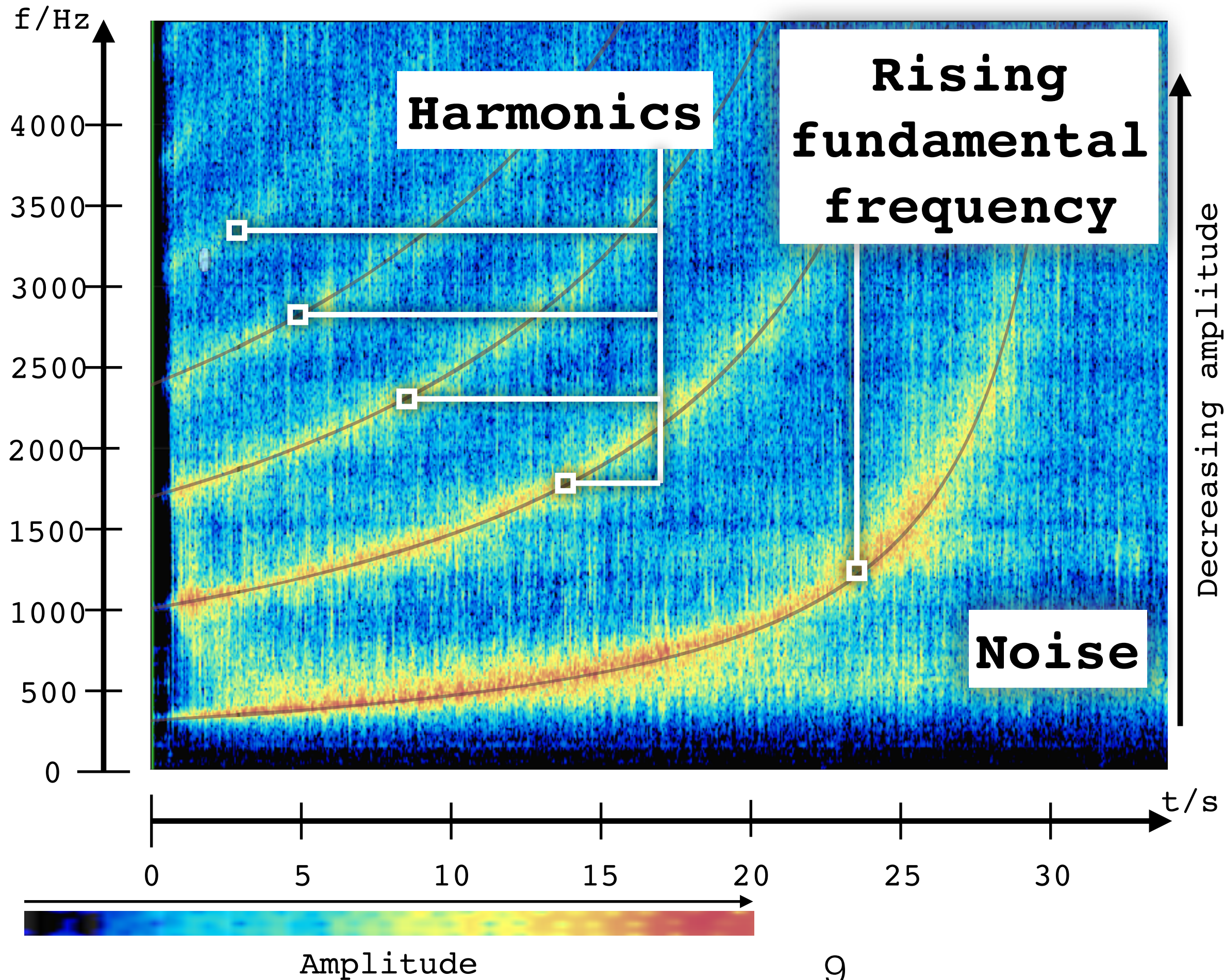
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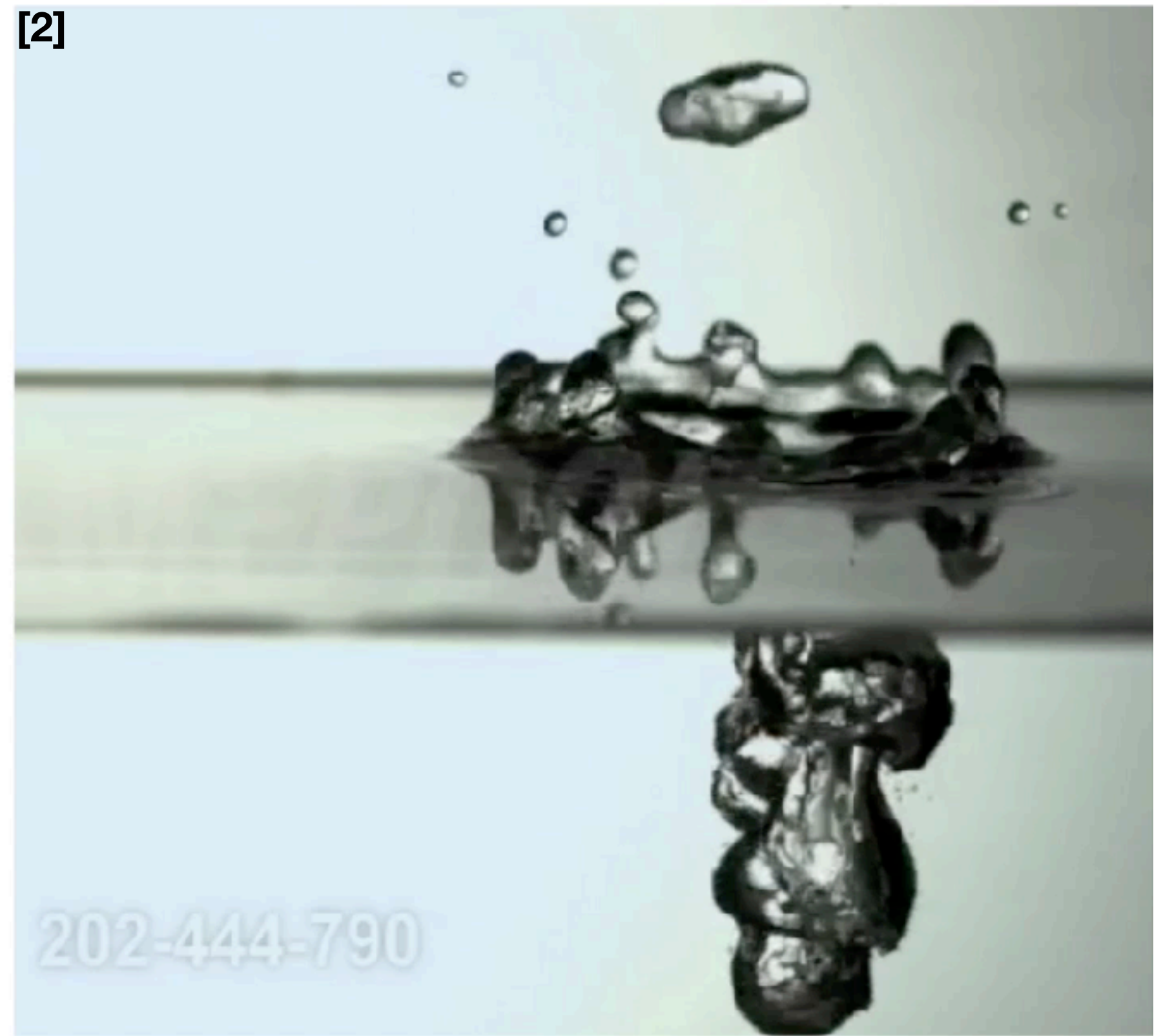
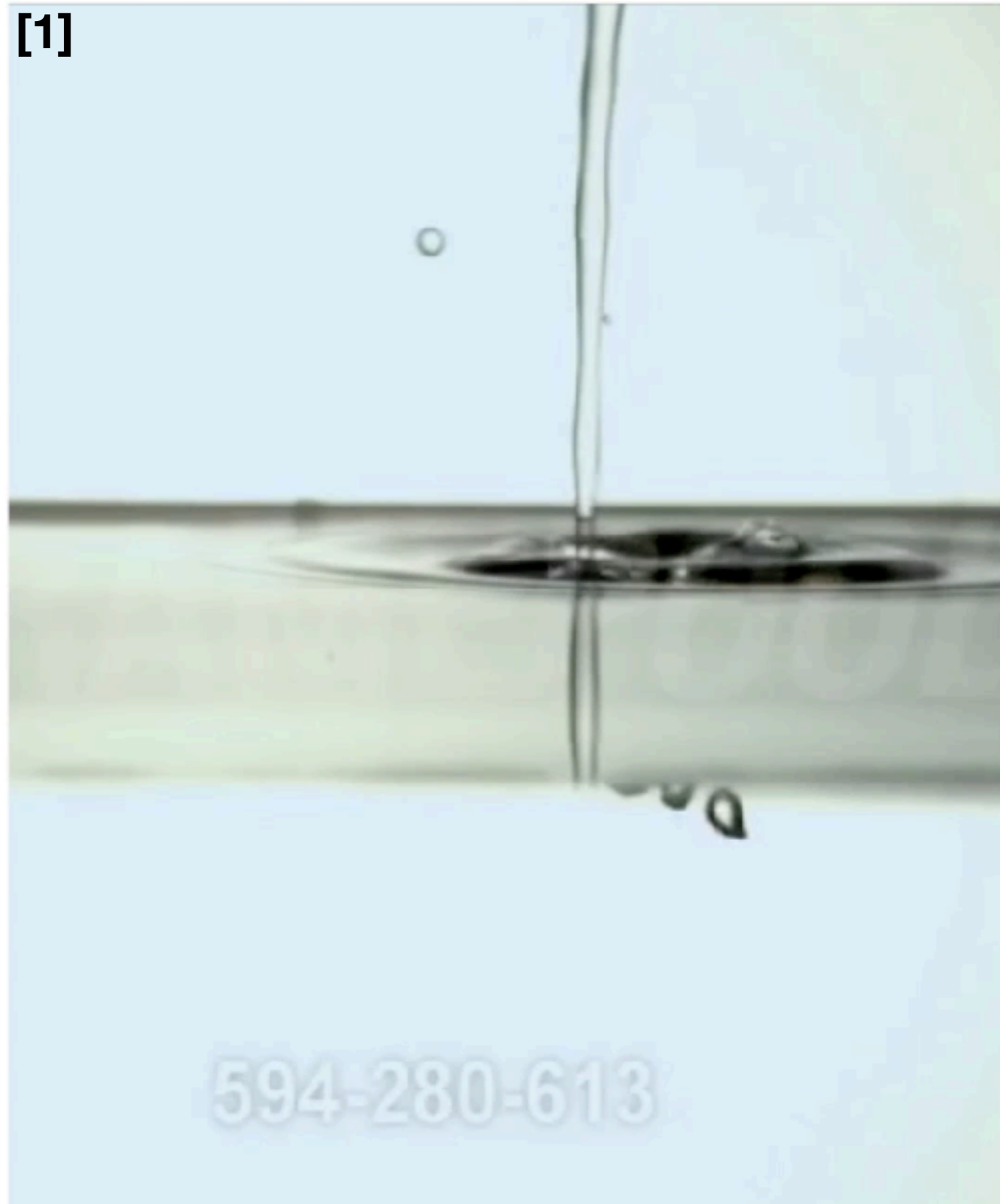


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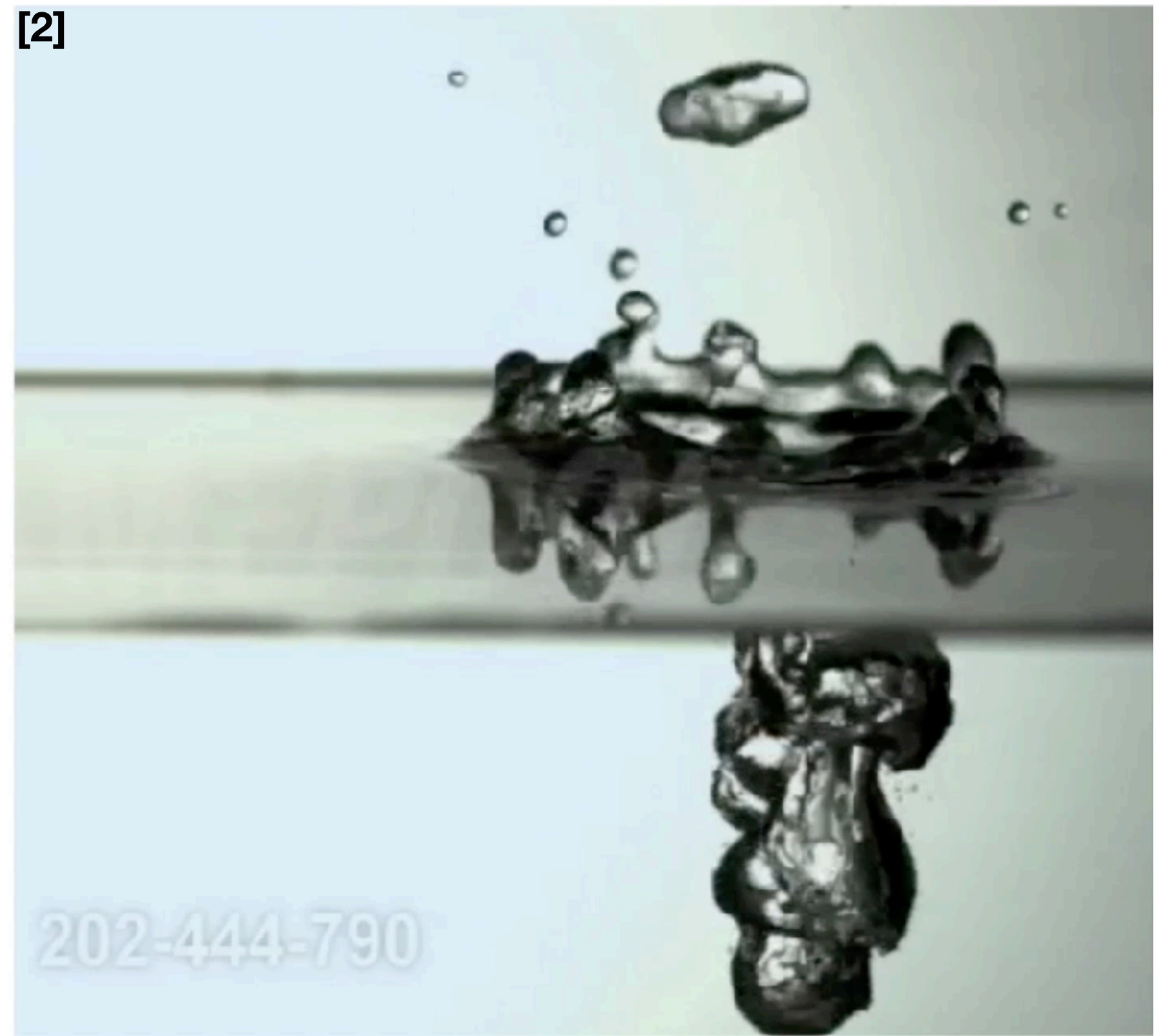
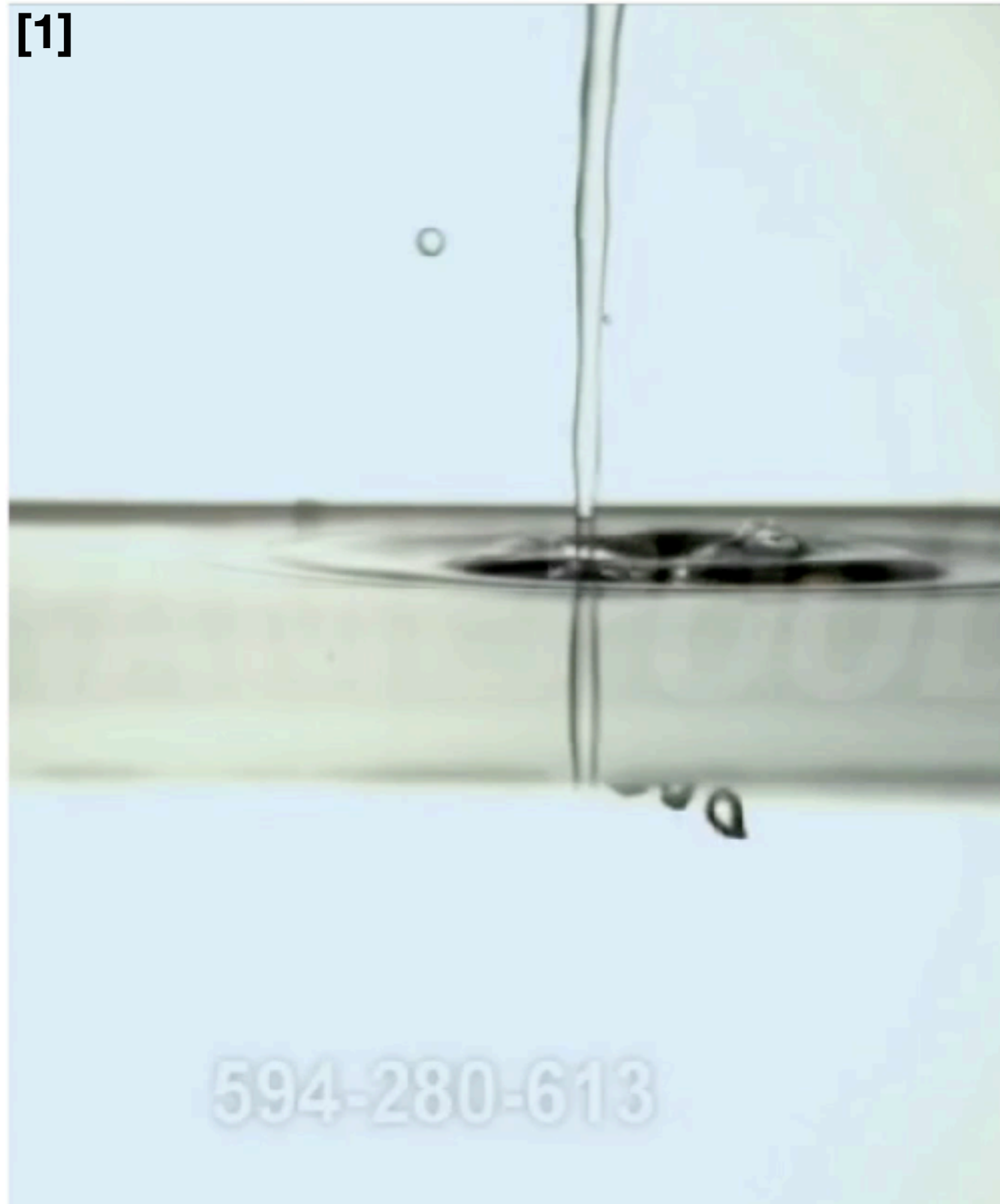
Jet-surface interactions



[1] <http://footage.framepool.com/de/shot/594280613-aufprall-auftreffen-glas-behaelter-wasserbecken-luftblase>

[2] <http://footage.framepool.com/de/shot/202444790-aufprall-auftreffen-glas-behaelter-wasserbecken-luftblase>

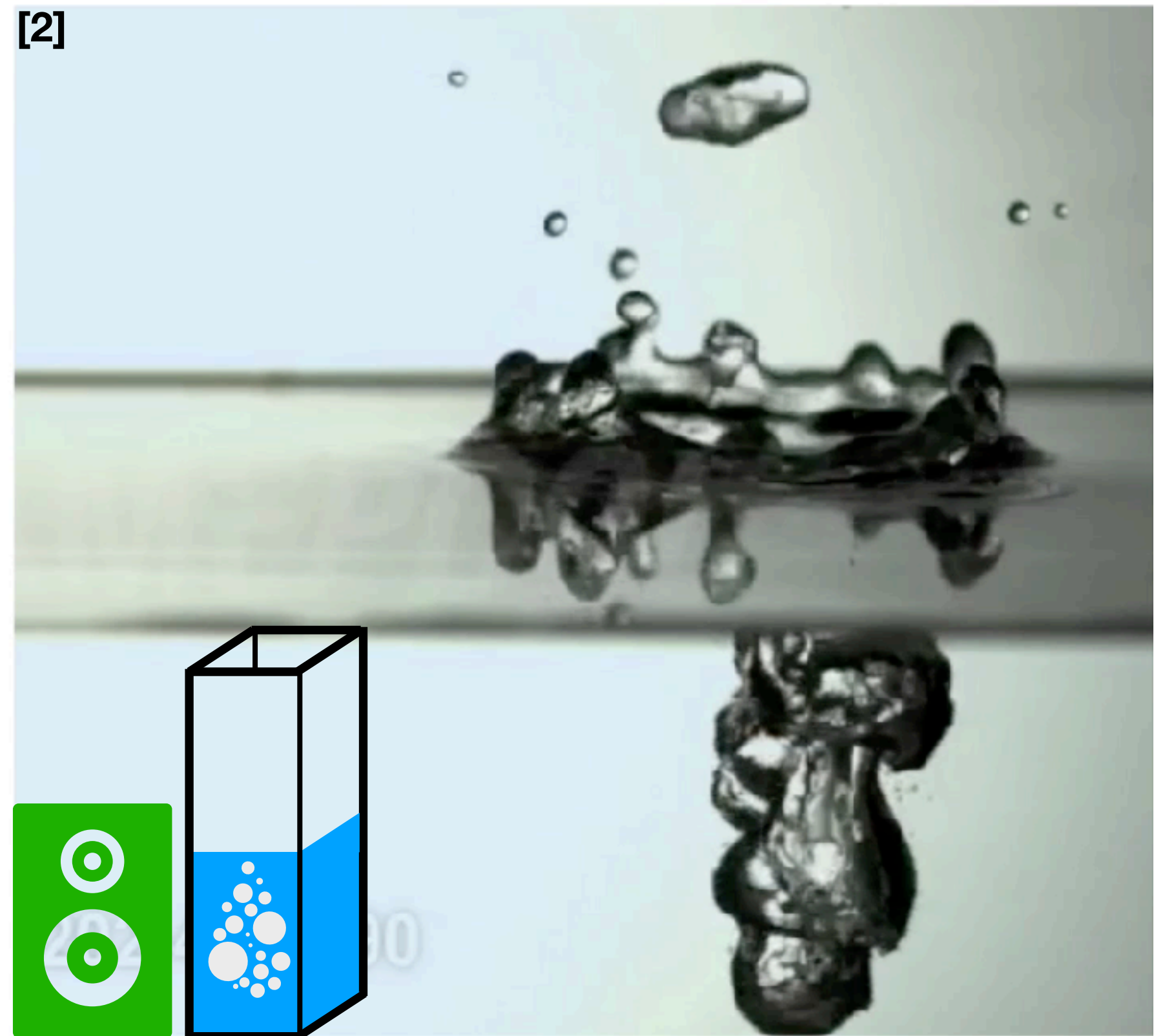
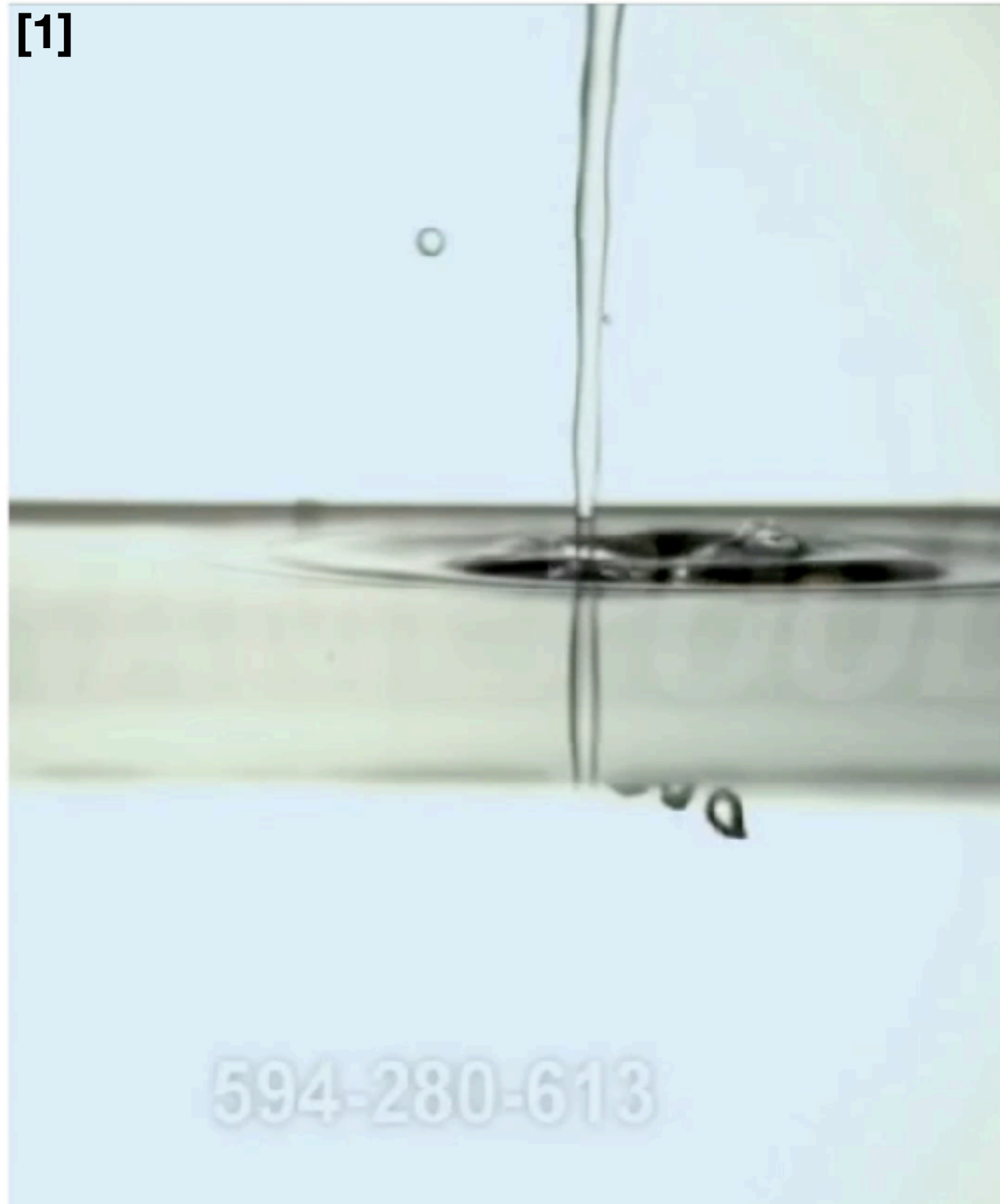
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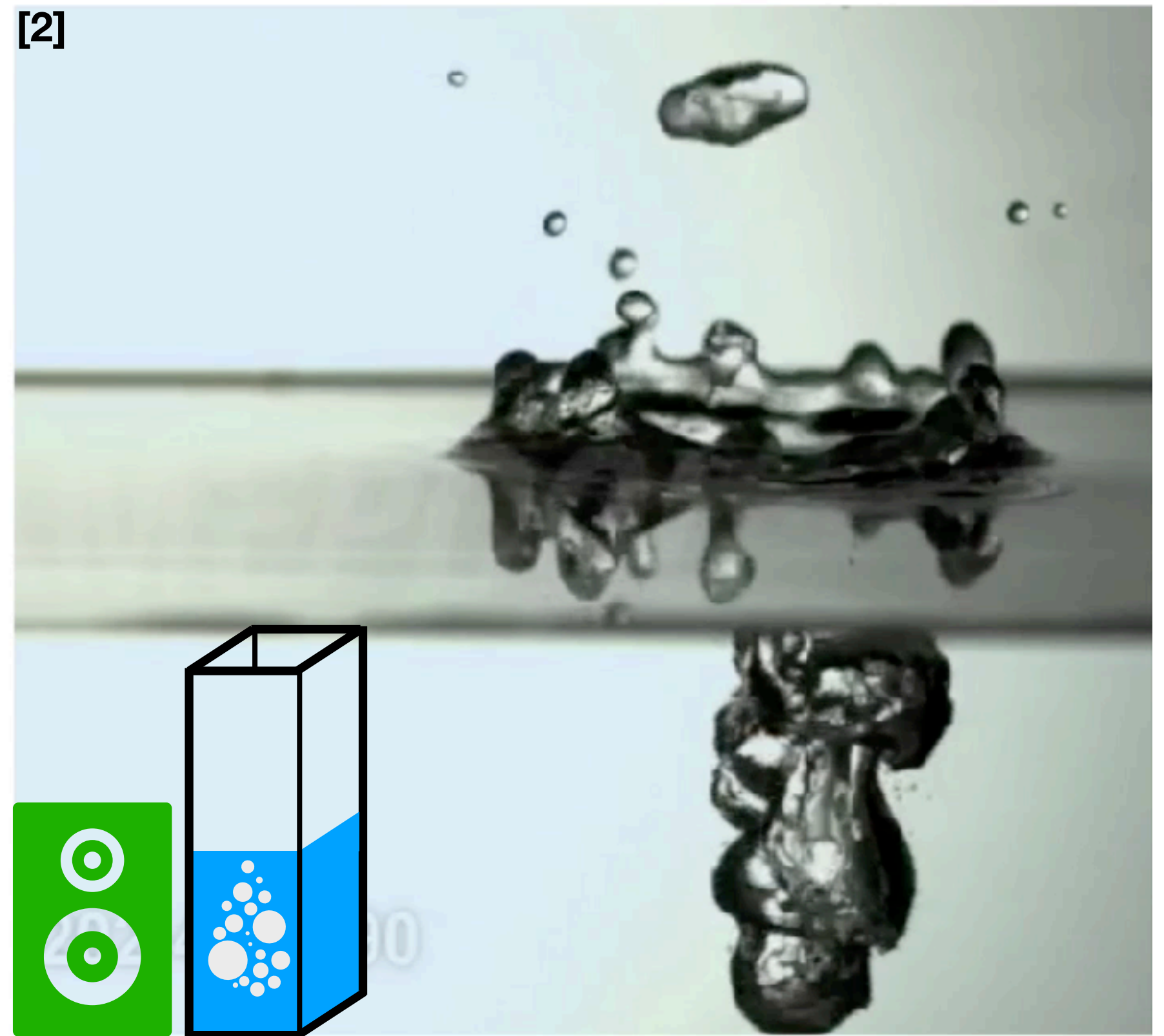
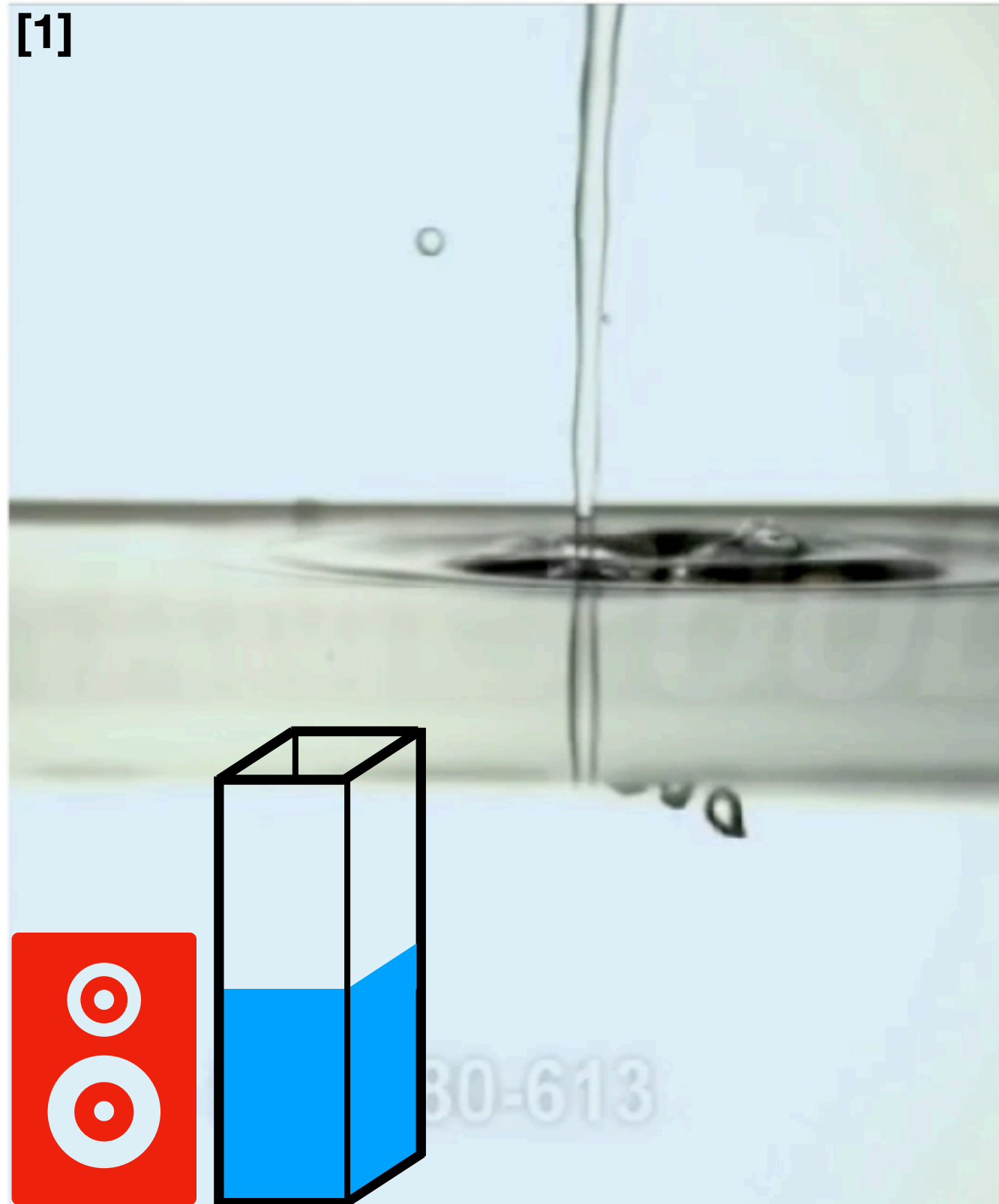


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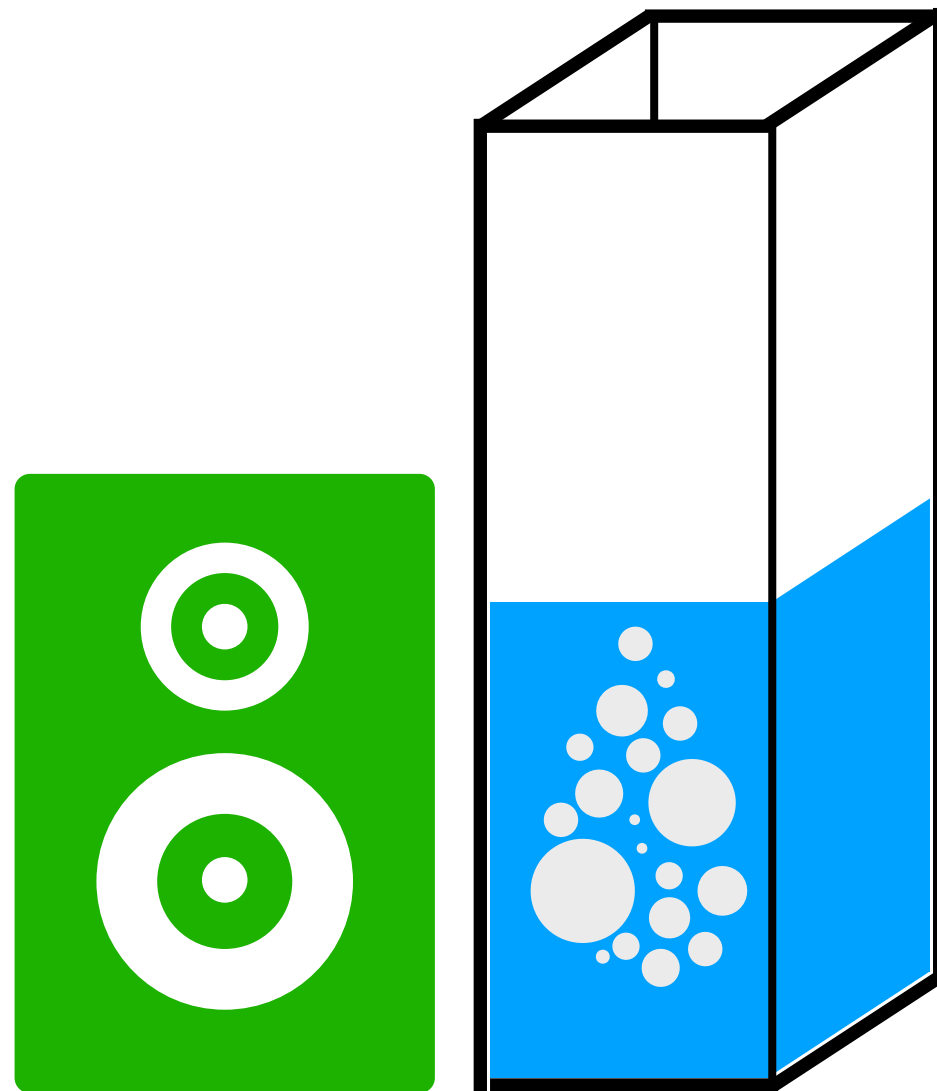


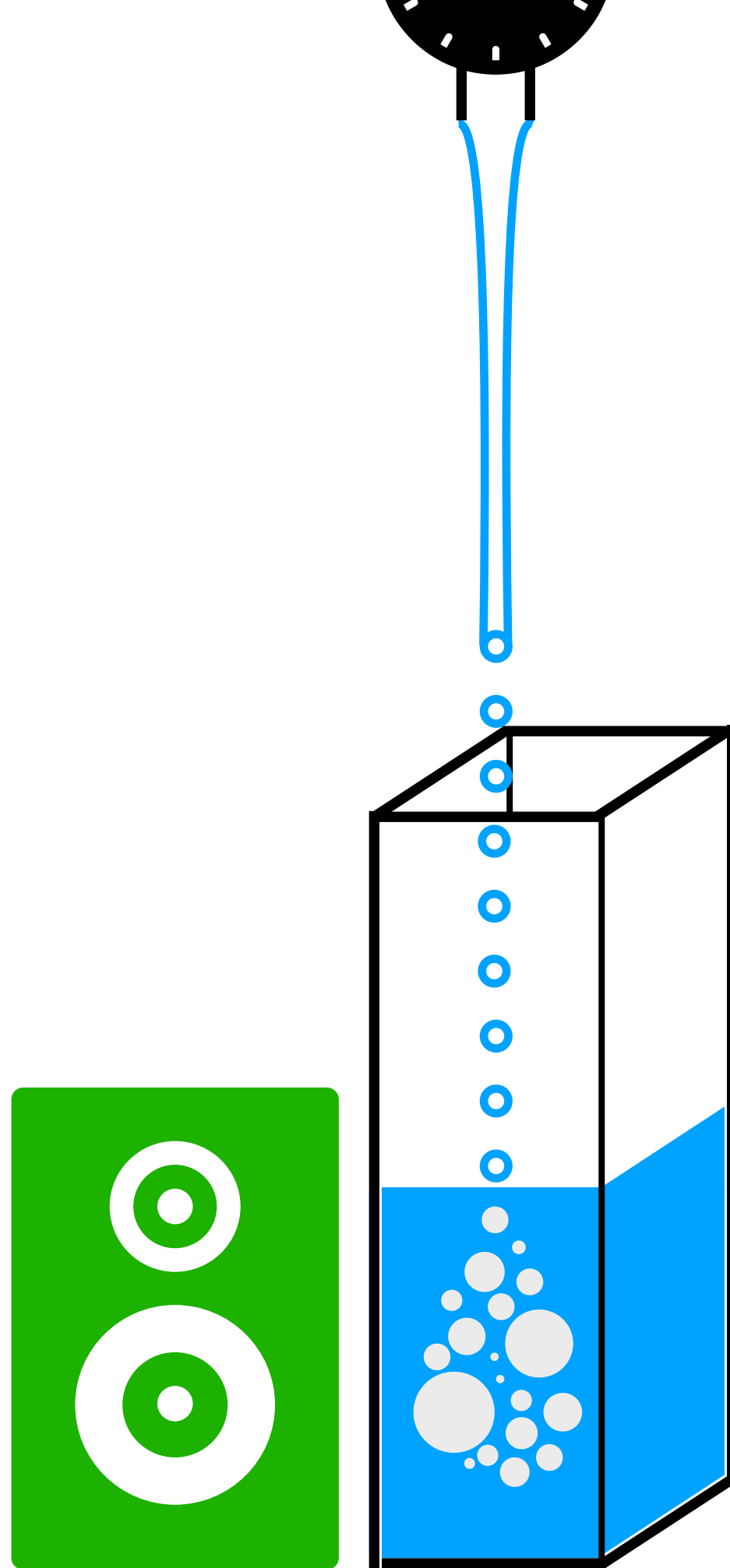
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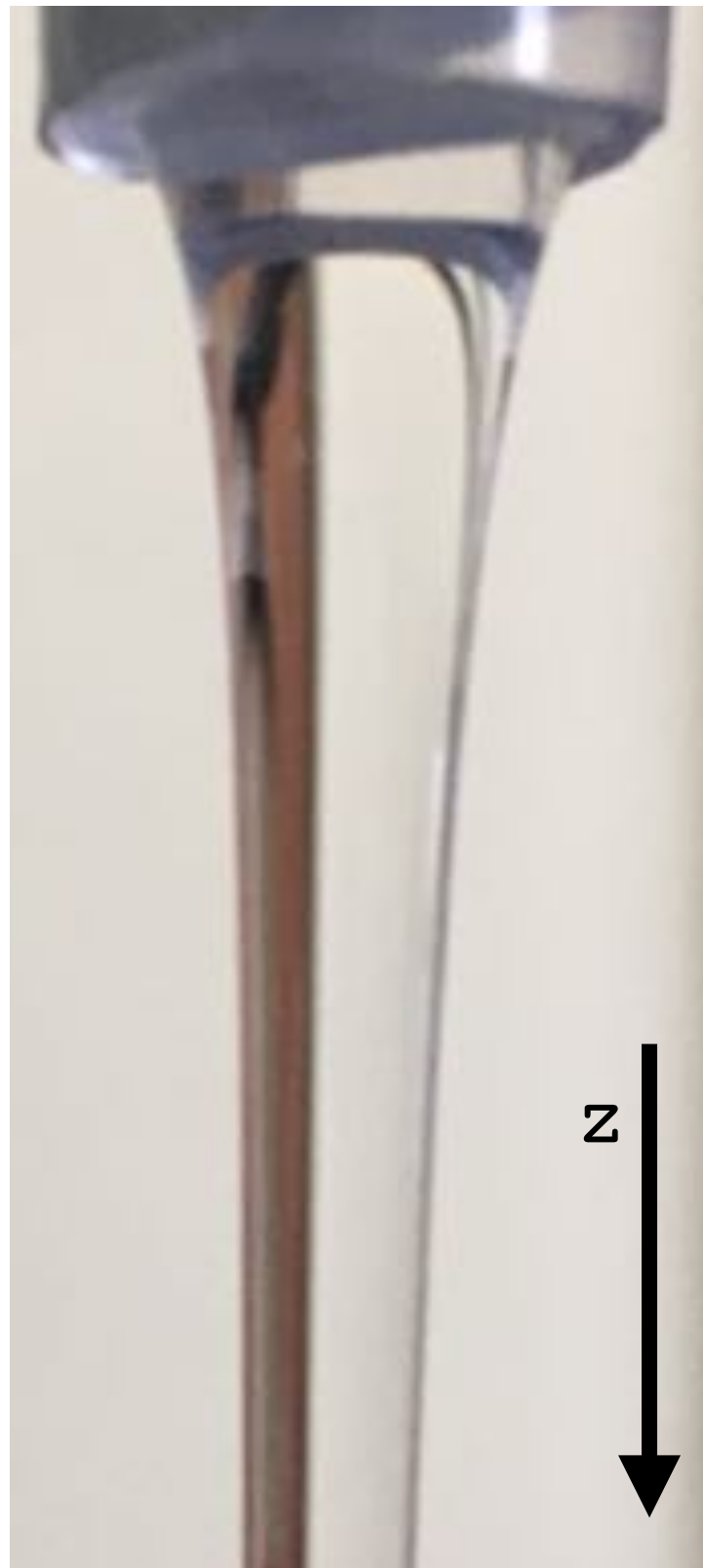


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[2] <http://footage.framepool.com/de/shot/202444790-aufprall-auftreffen-glas-behaelter-wasserbecken-luftblase>

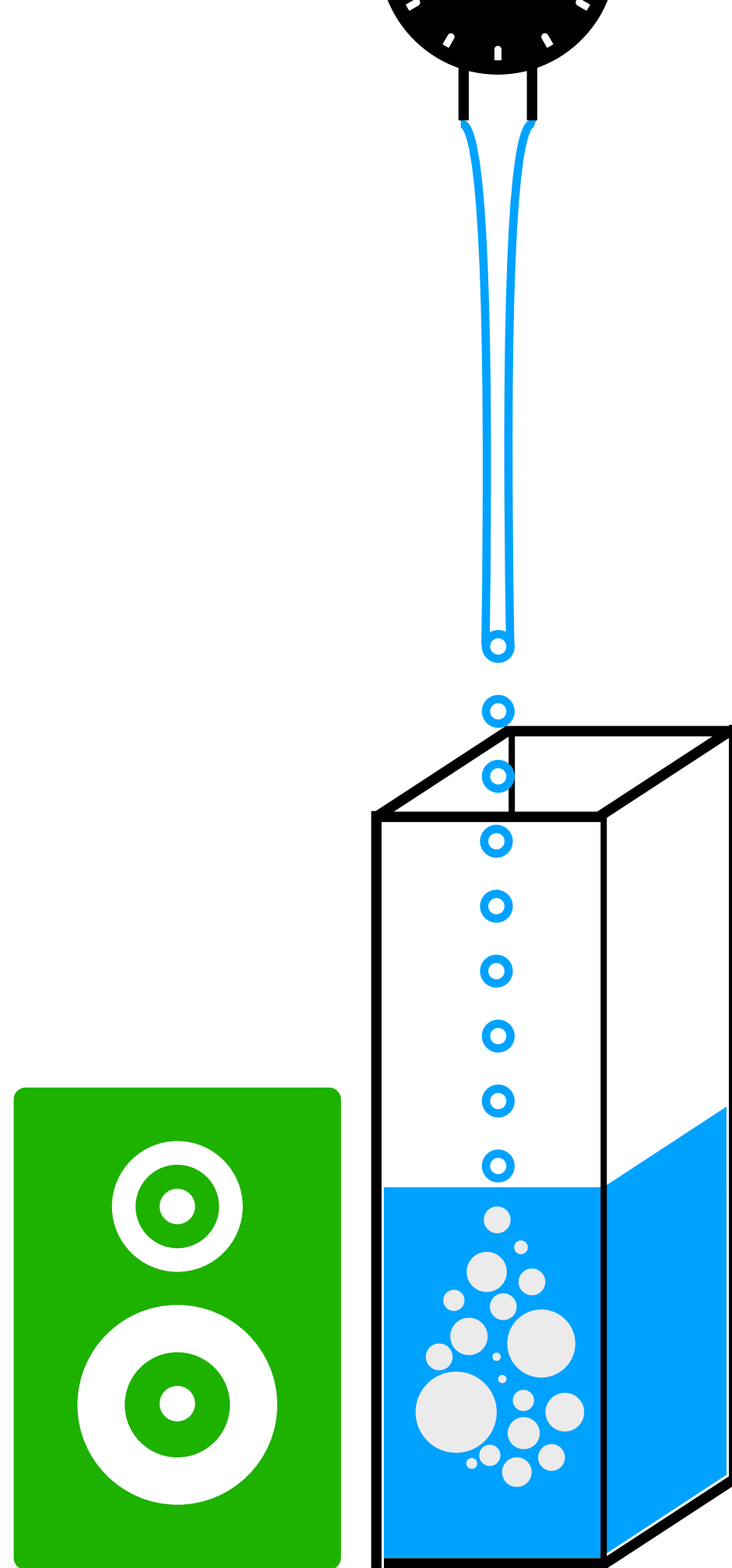


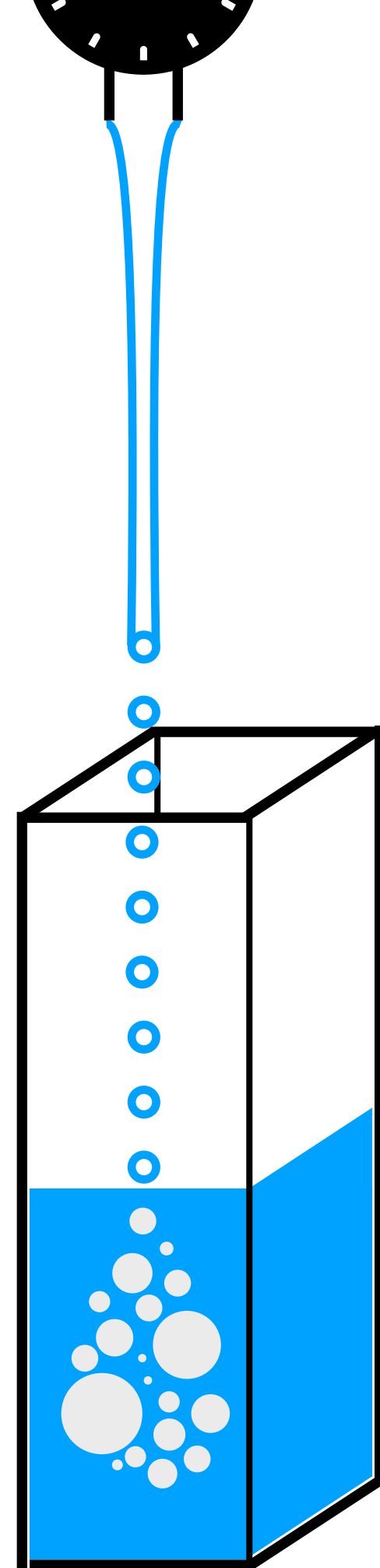
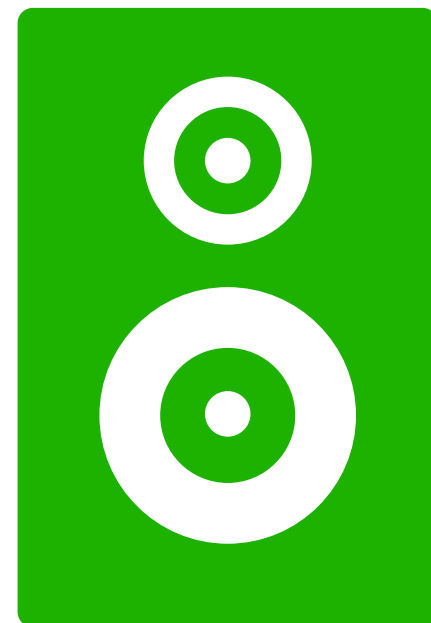
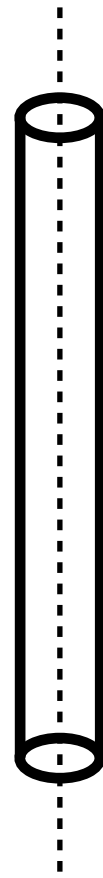
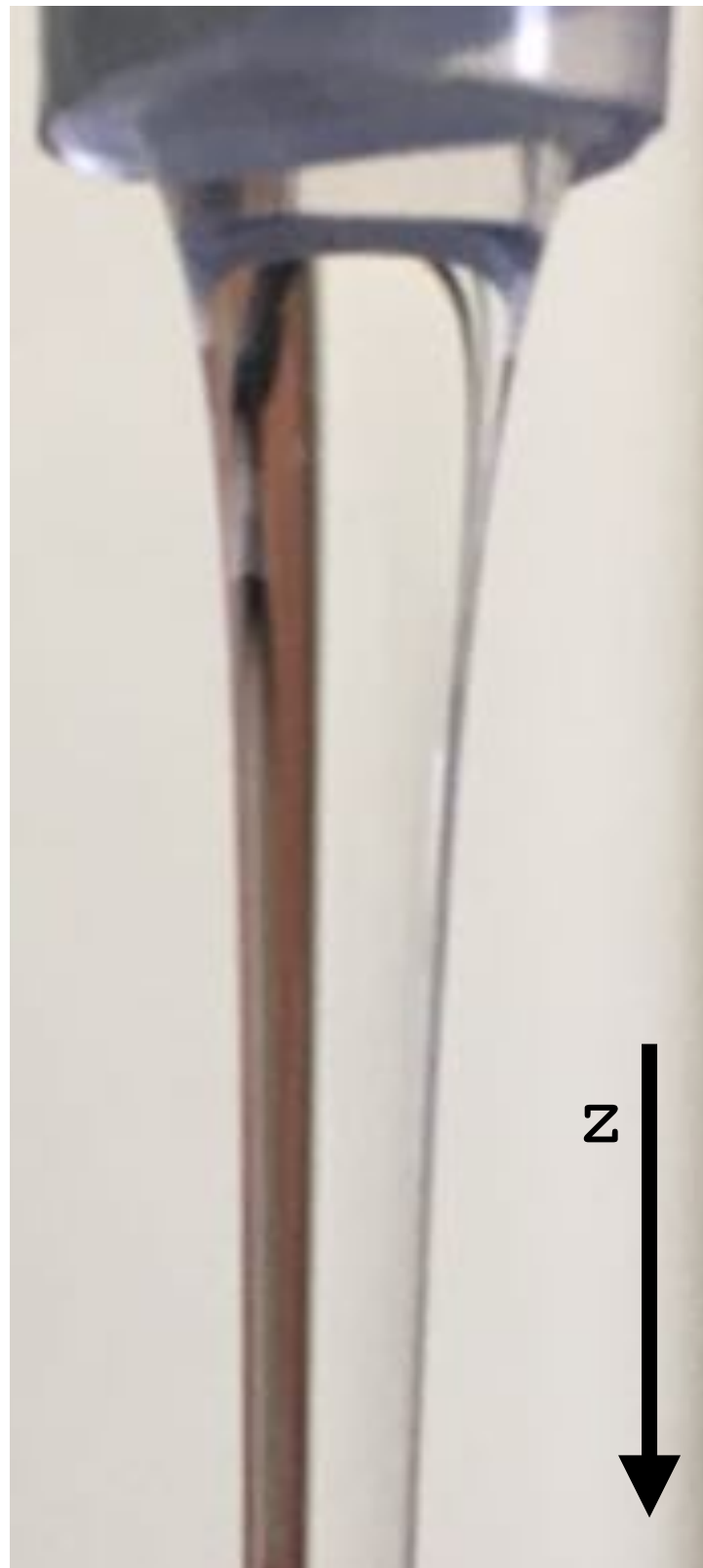




$z \uparrow \Rightarrow v \uparrow \Rightarrow r \downarrow$

$Q = \text{const.}$



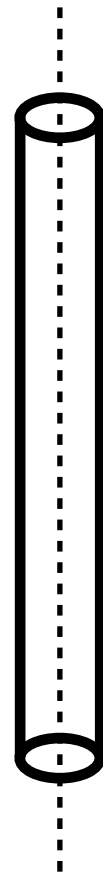


$z \uparrow \Rightarrow v \uparrow \Rightarrow r \downarrow$

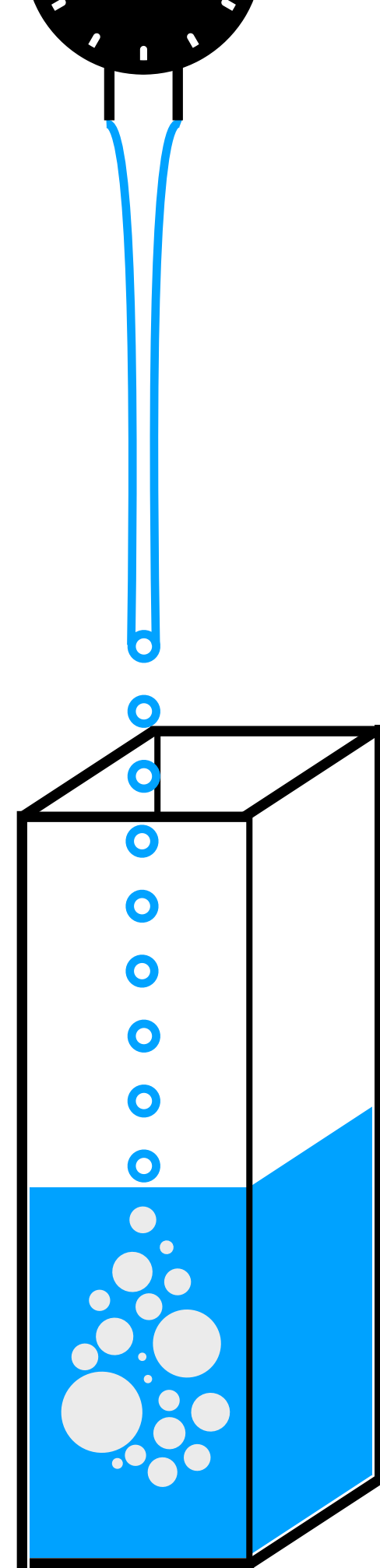
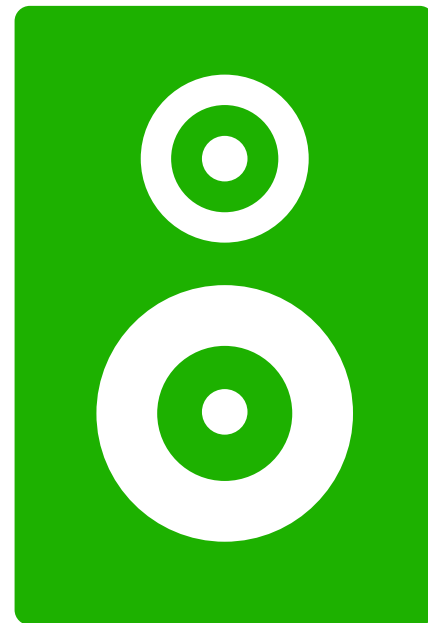
$Q = \text{const.}$



z



$$\lambda > \pi \cdot r(z)$$

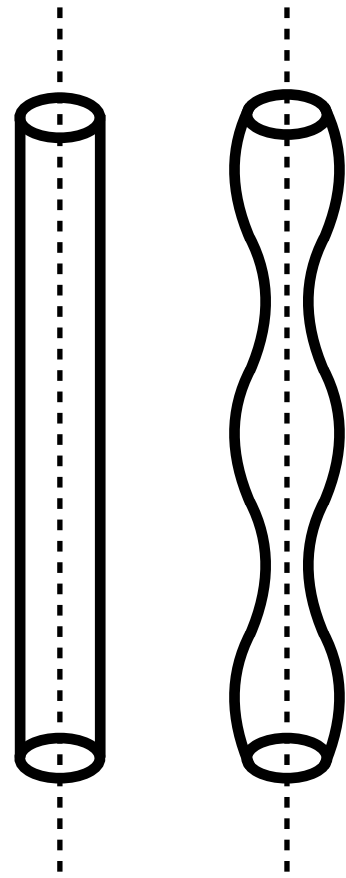


$$z \uparrow \Rightarrow v \uparrow \Rightarrow r \downarrow$$

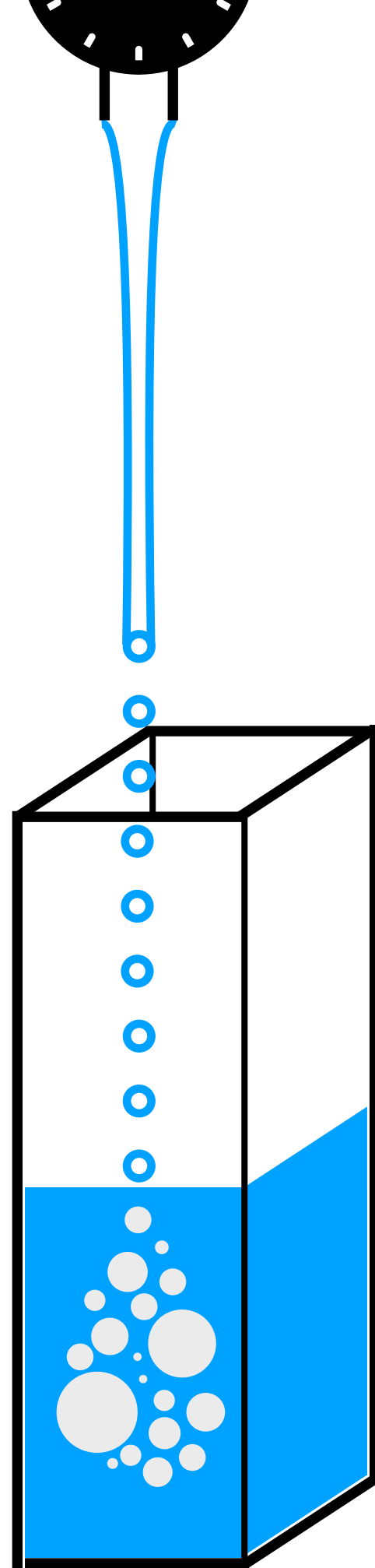
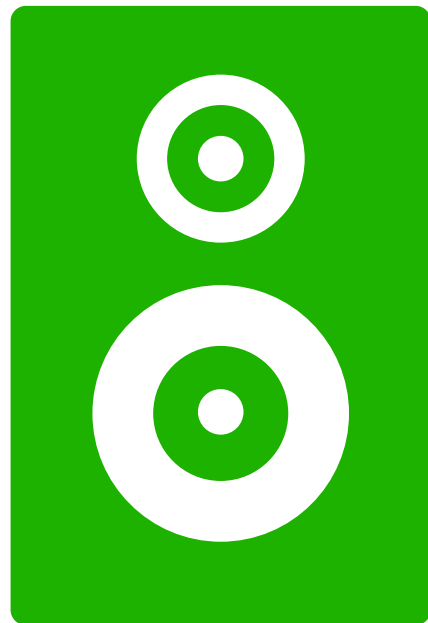
$$Q = \text{const.}$$



z

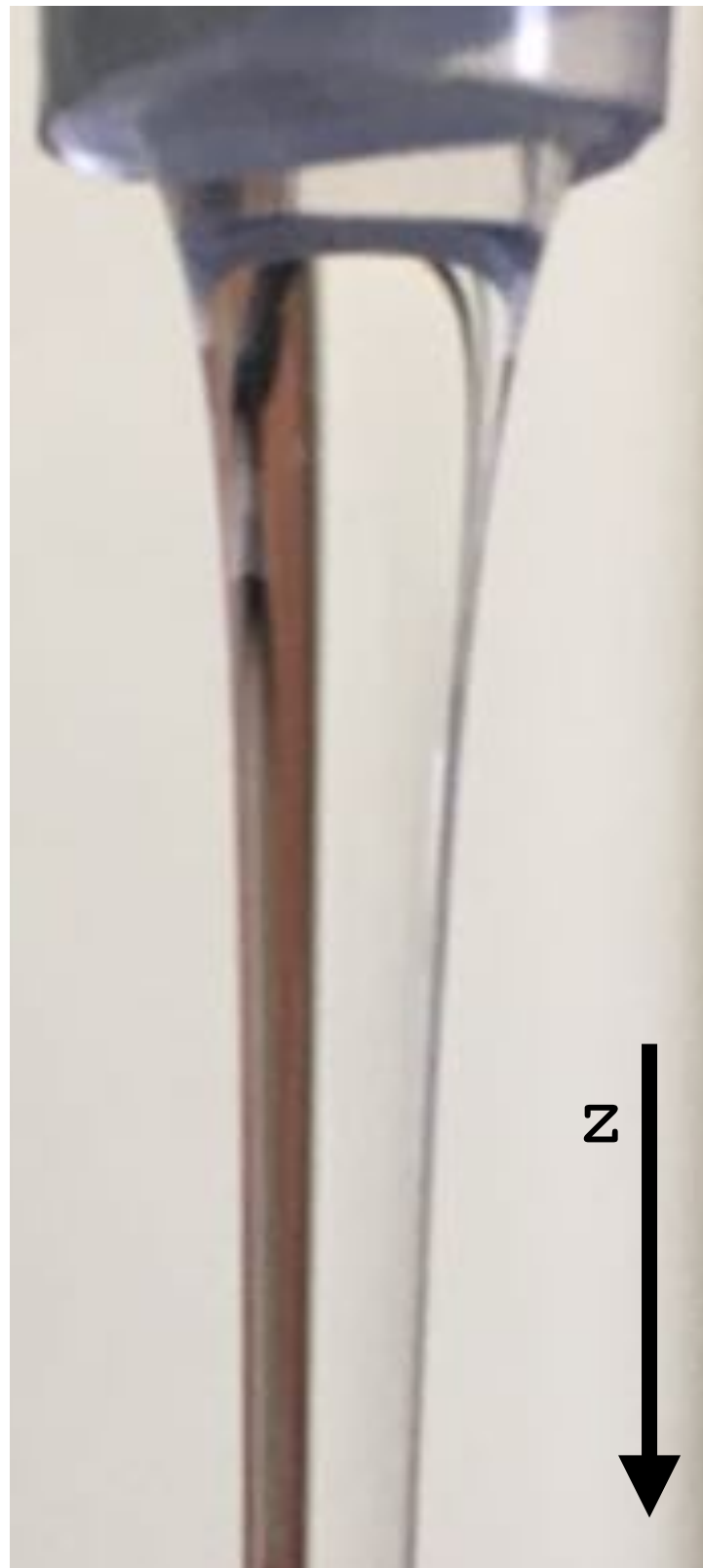


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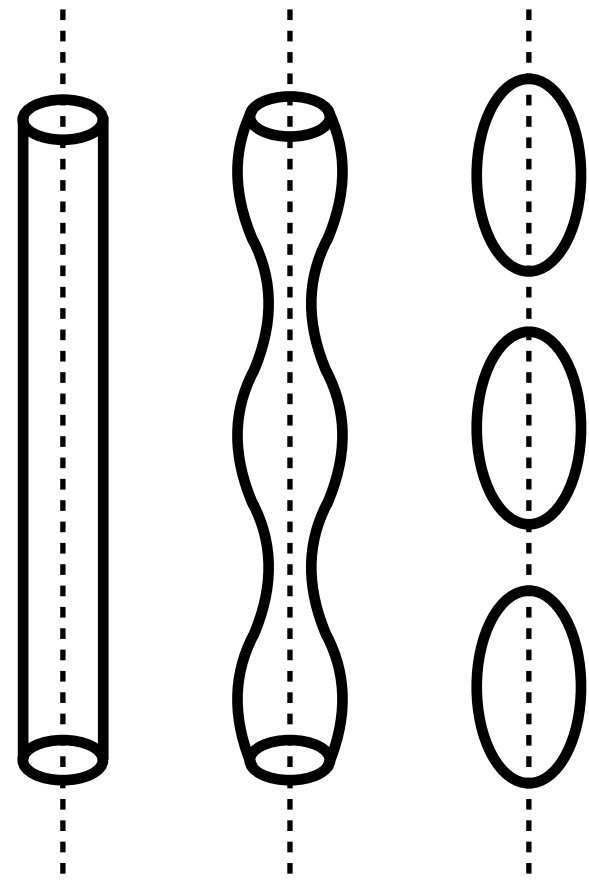


$$z \uparrow \Rightarrow v \uparrow \Rightarrow r \downarrow$$

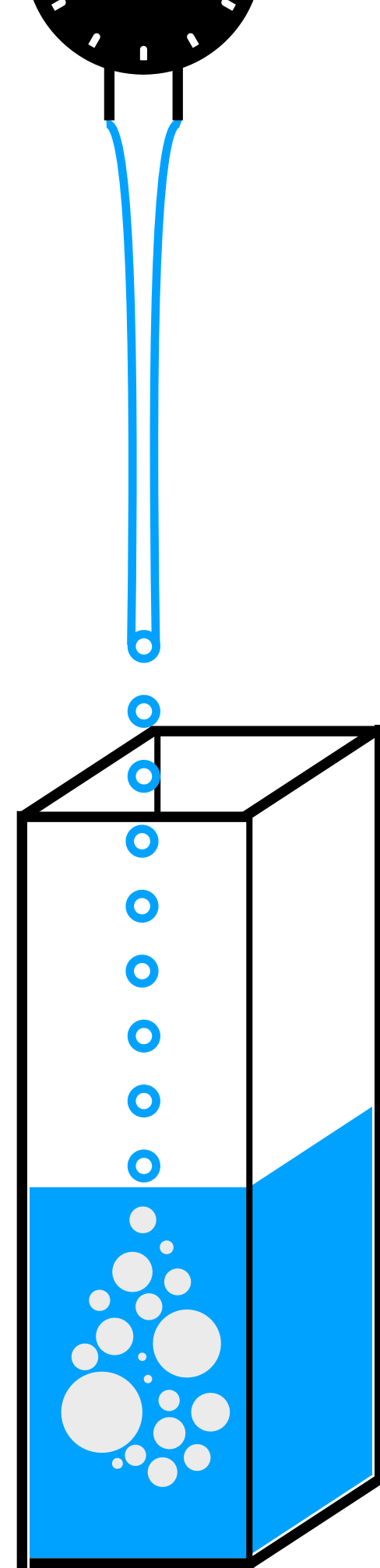
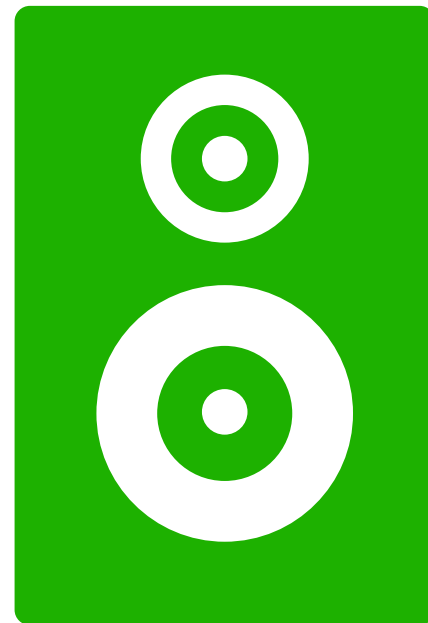
$$Q = \text{const.}$$



z

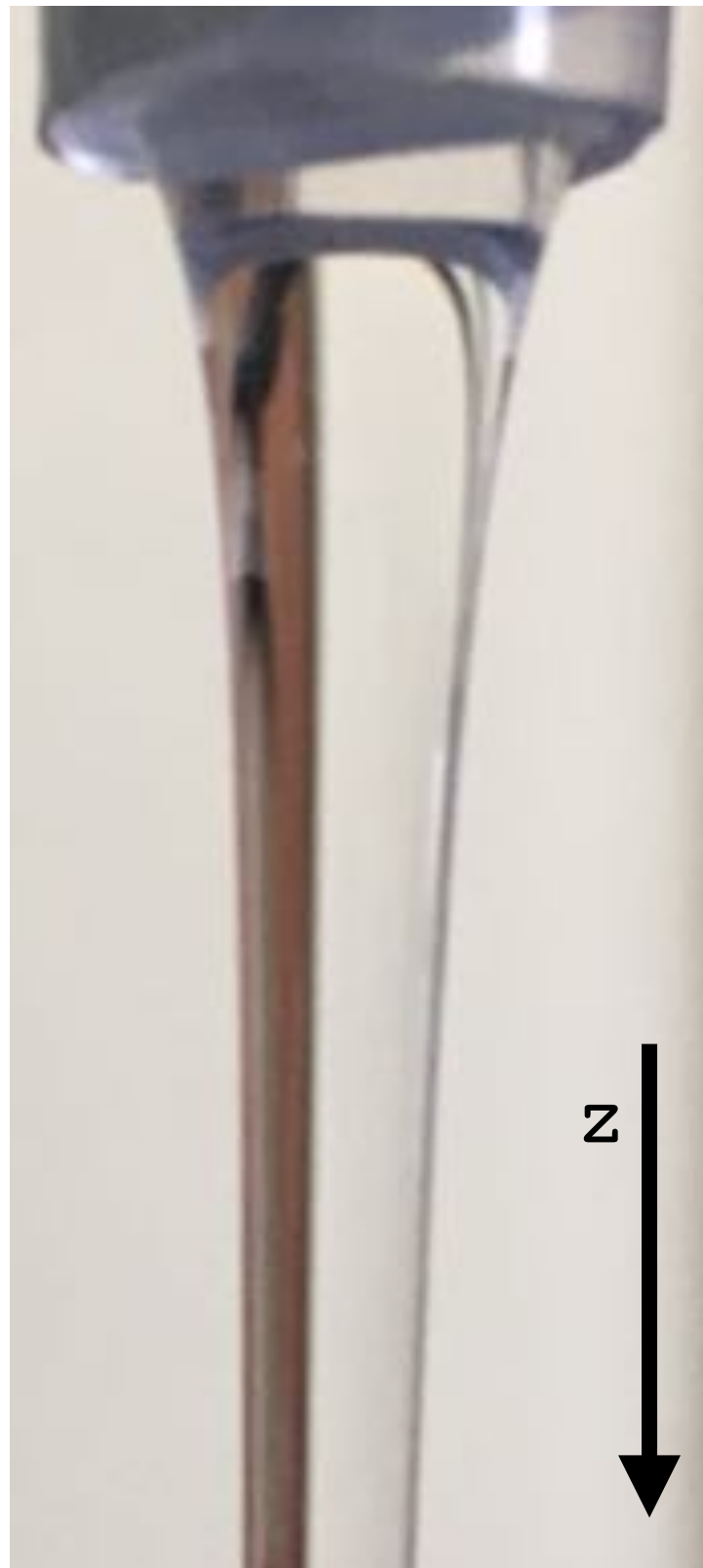


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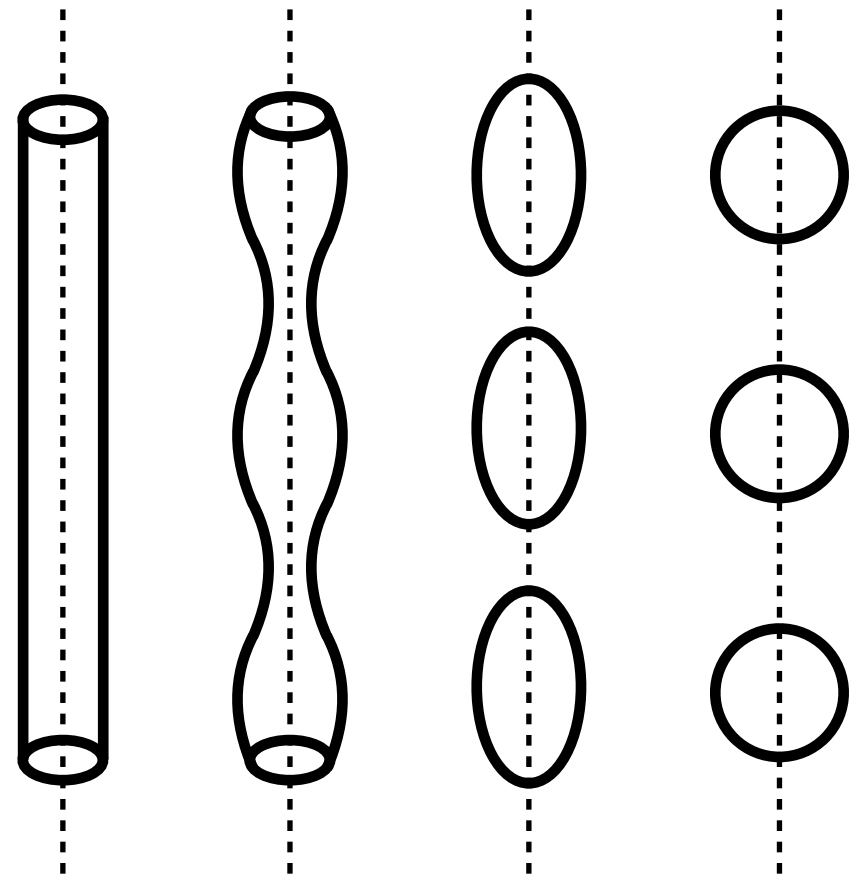


$$z \uparrow \Rightarrow v \uparrow \Rightarrow r \downarrow$$

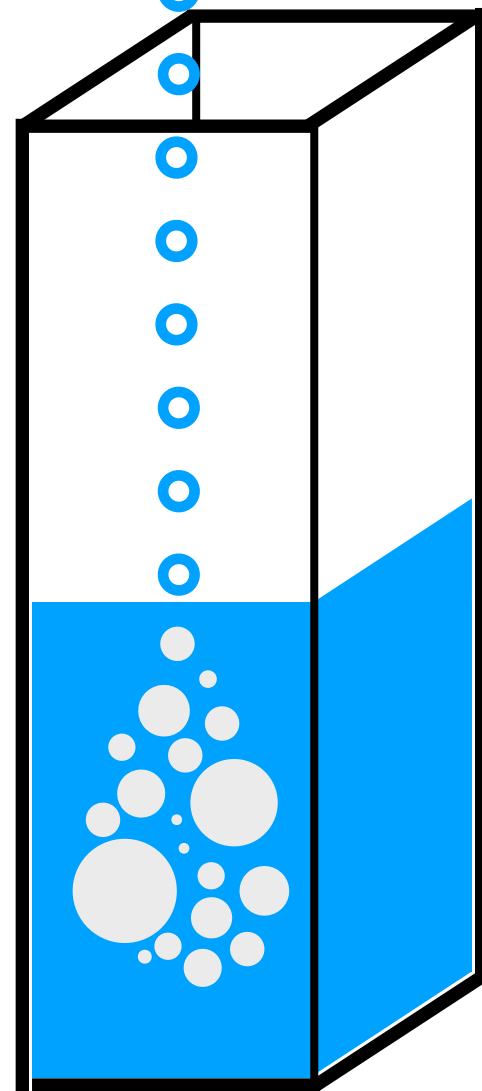
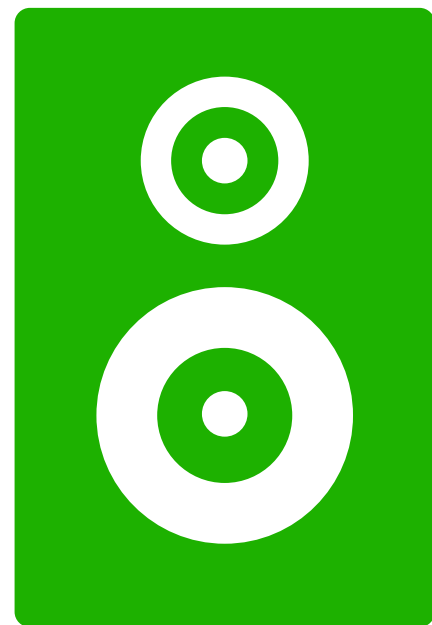
$$Q = \text{const.}$$



z



$$\lambda > \pi \cdot r(z)$$

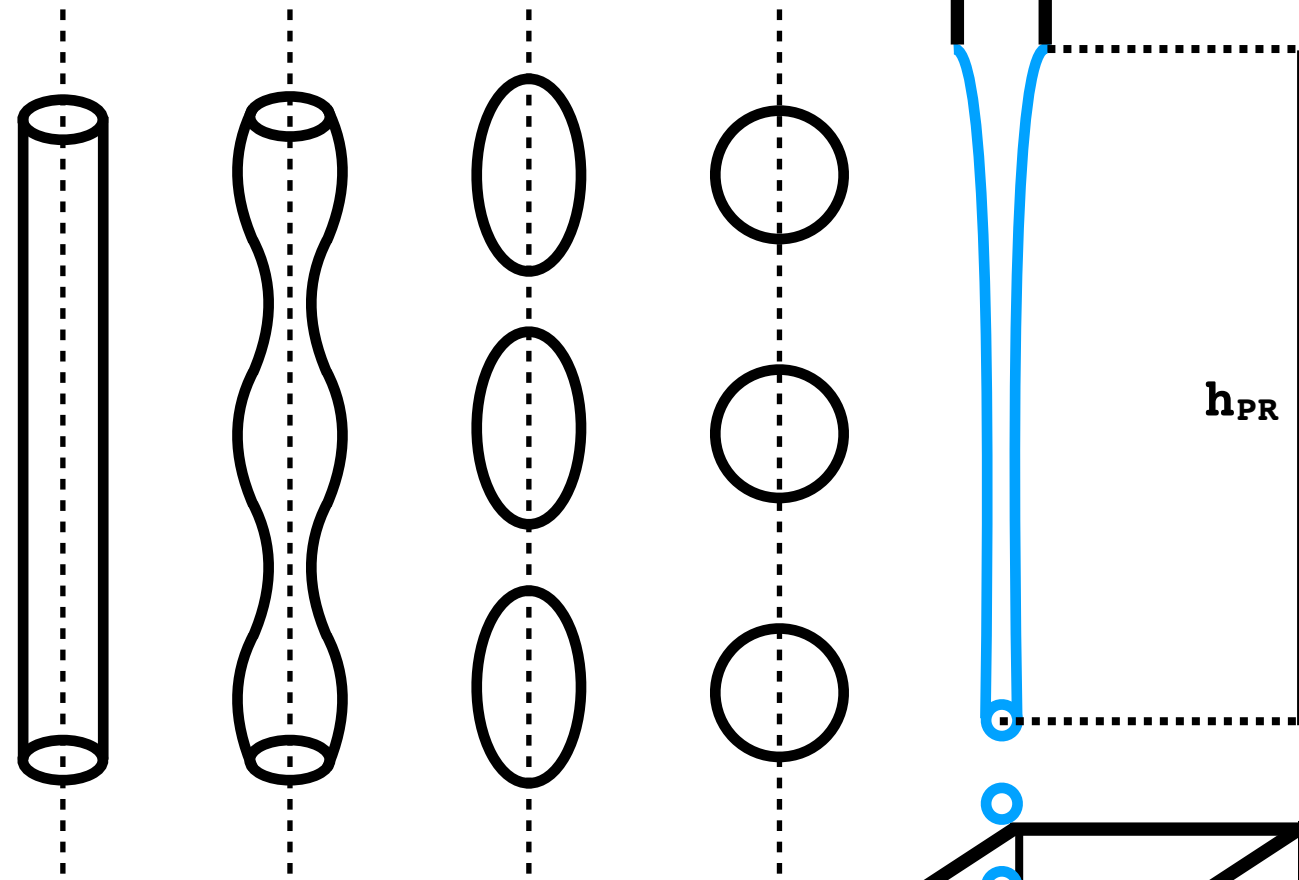


$z \uparrow \Rightarrow v \uparrow \Rightarrow r \downarrow$

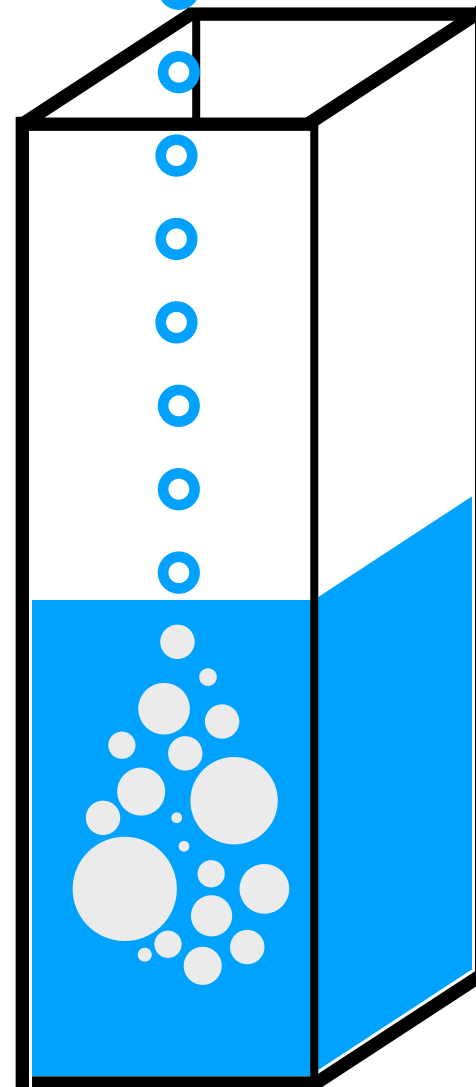
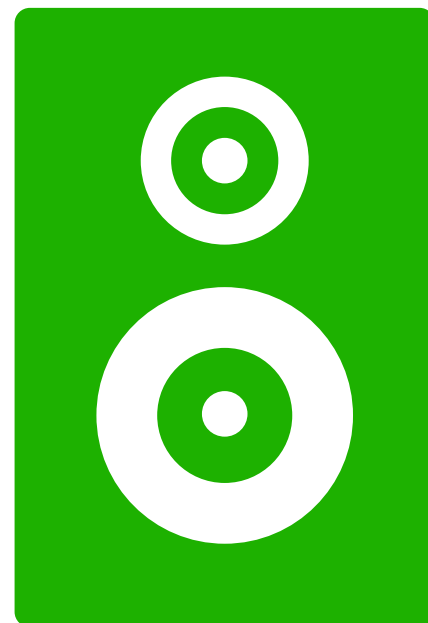
$Q = \text{const.}$



z

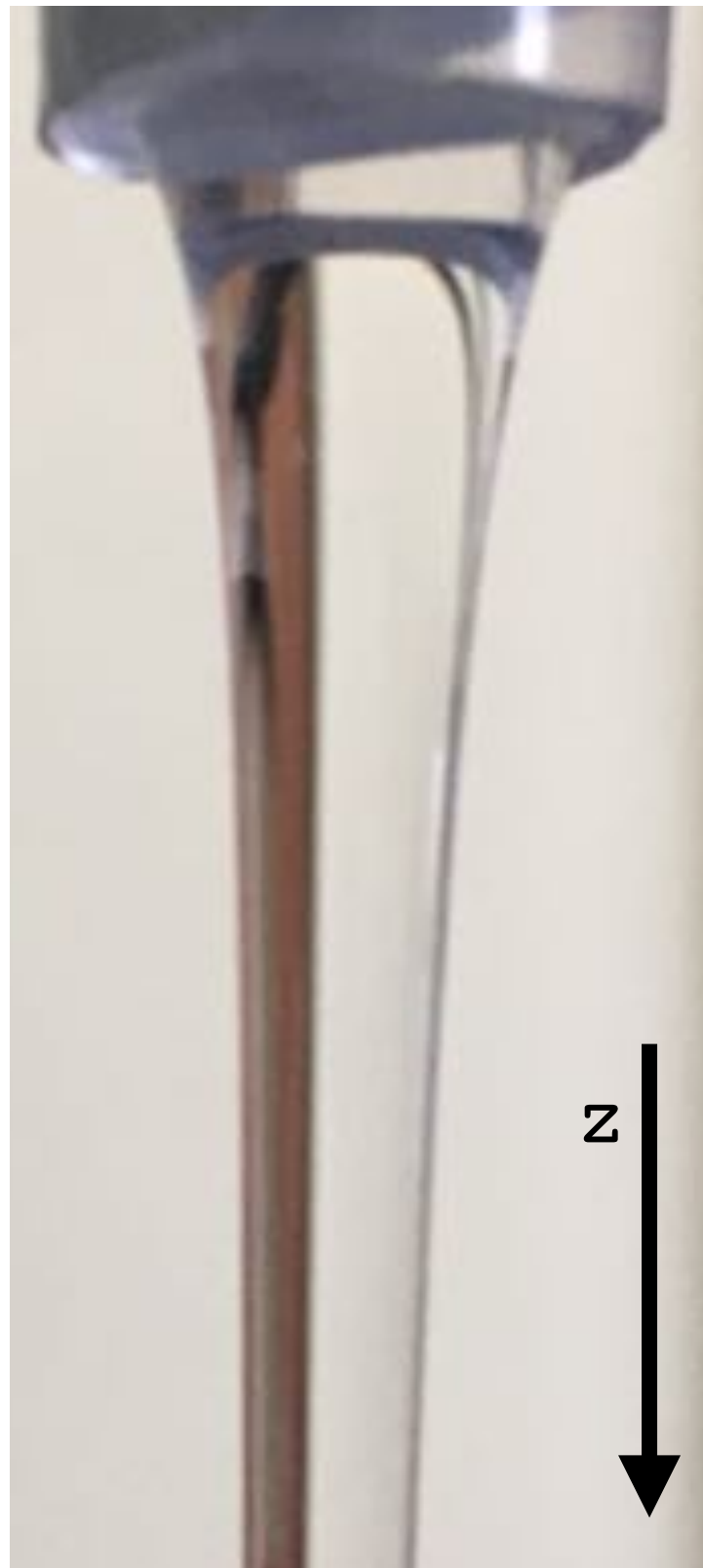


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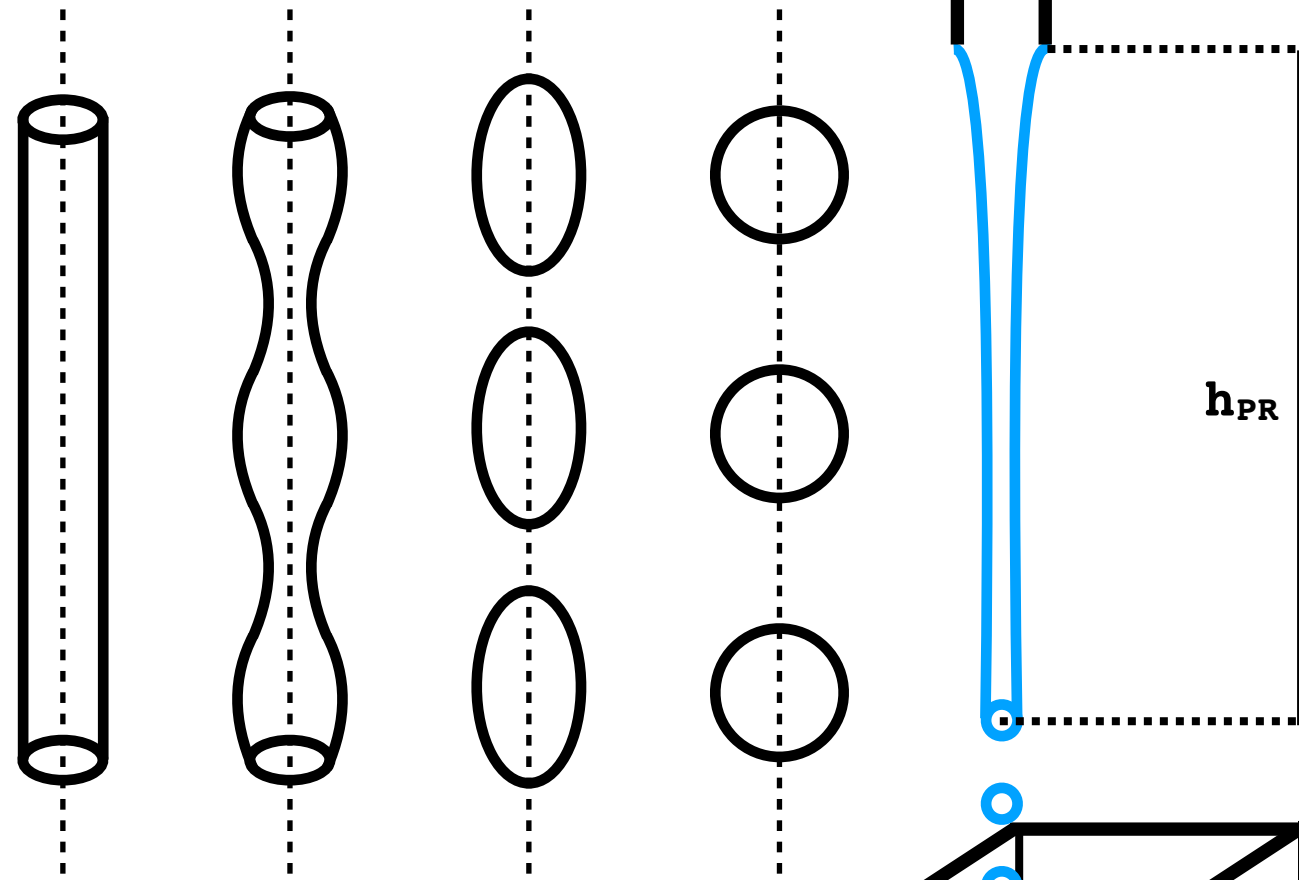
$$Q = \text{const.}$$



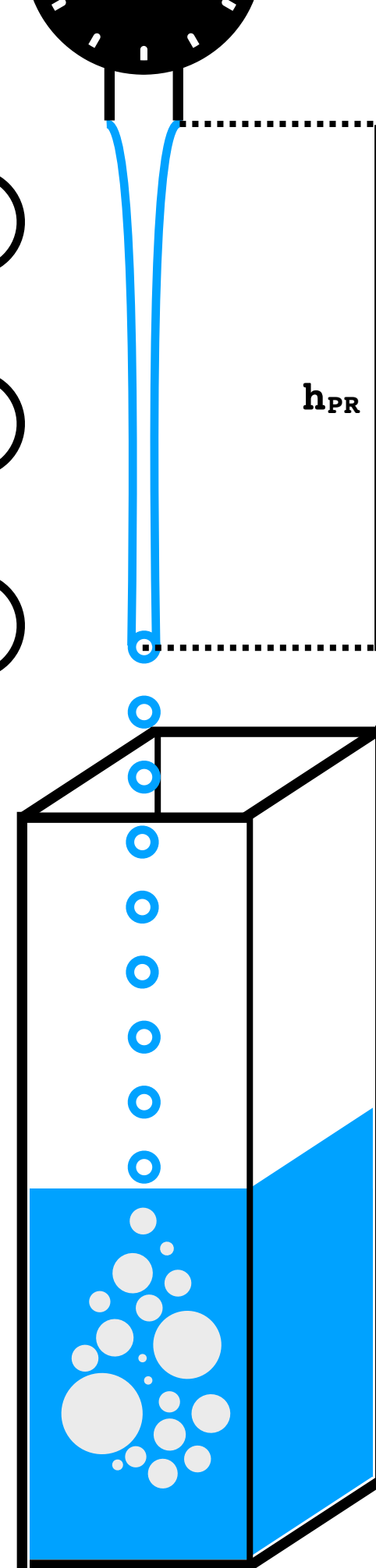
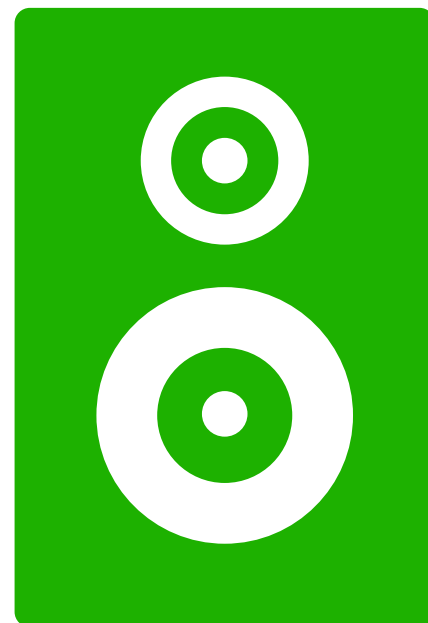
z

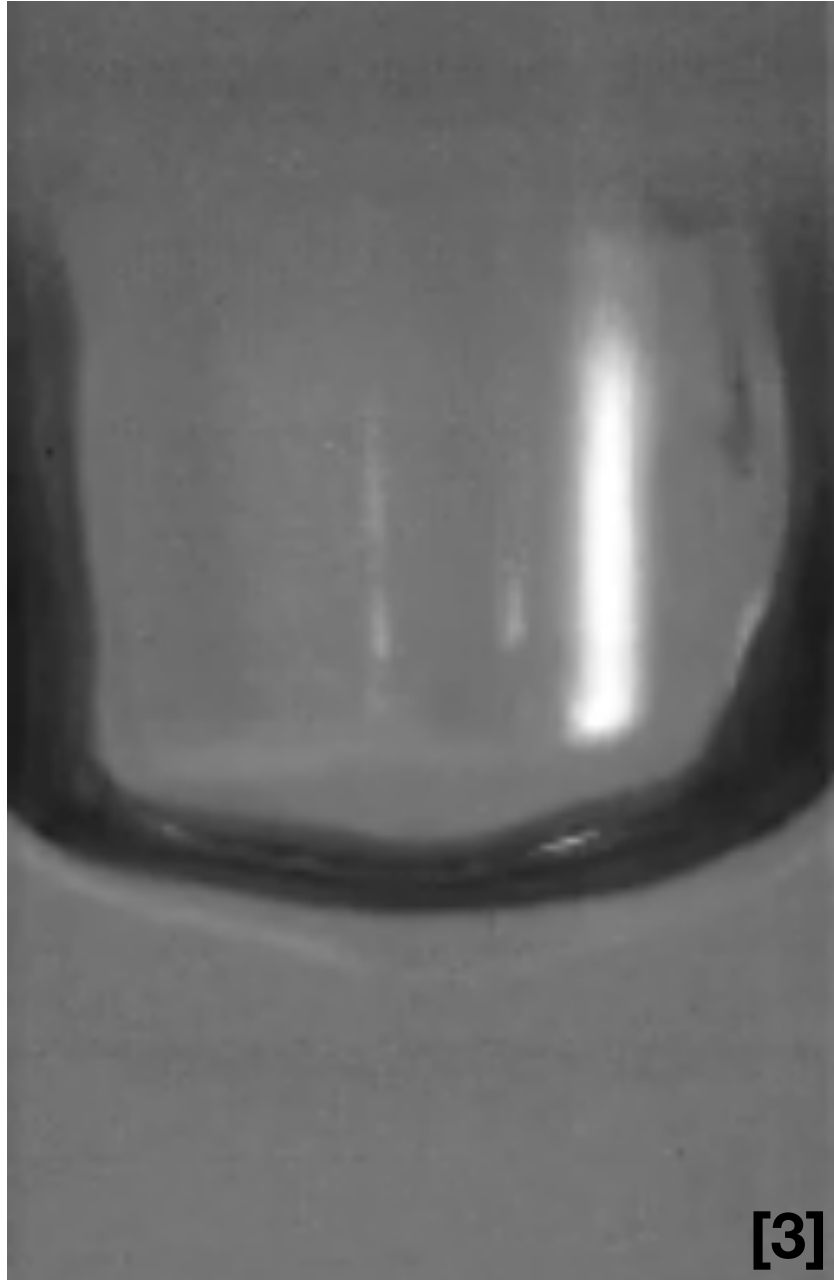
$z \uparrow \Rightarrow v \uparrow \Rightarrow r \downarrow$

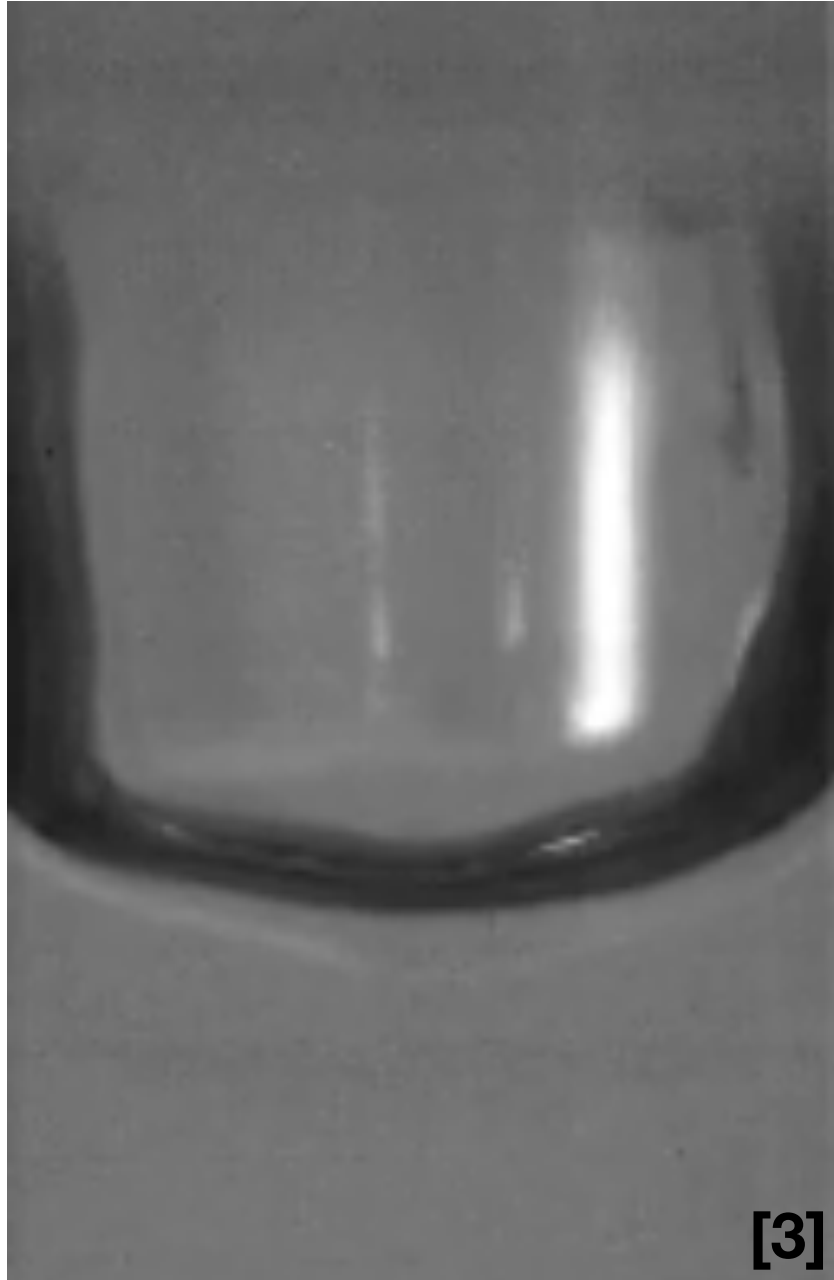
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$$\lambda > \pi \cdot r(z)$$

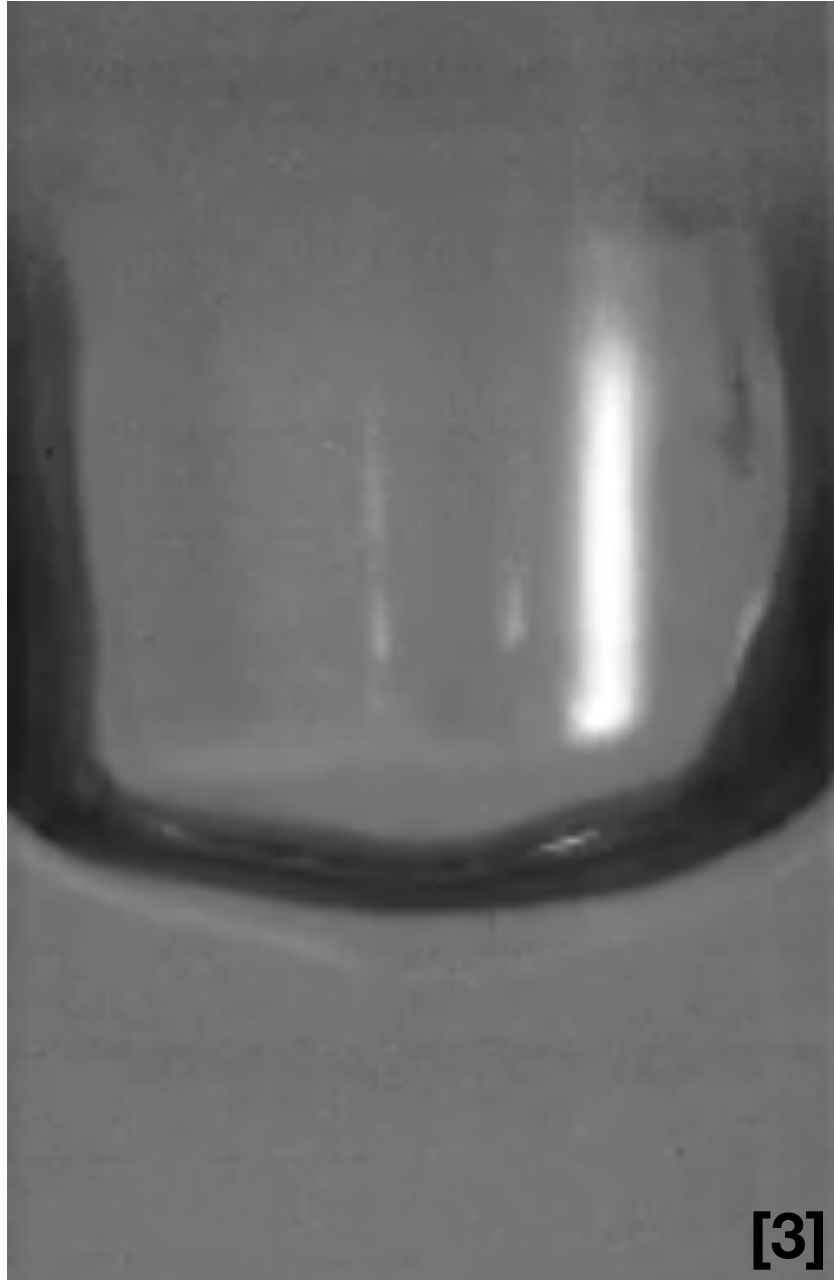








Minnaert frequency

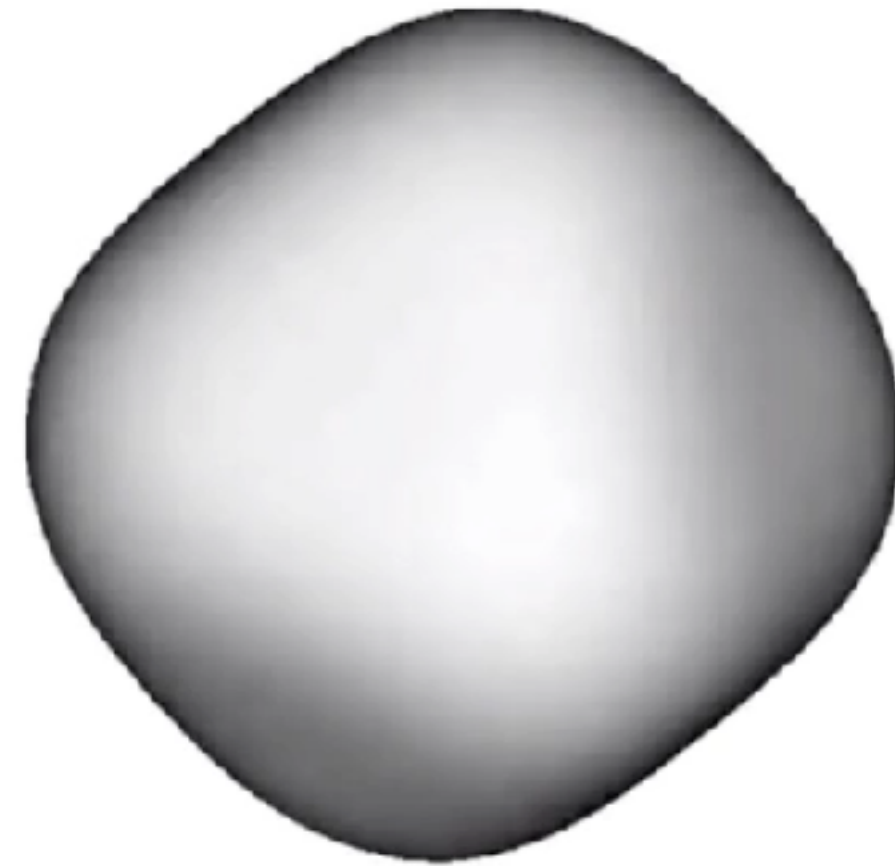
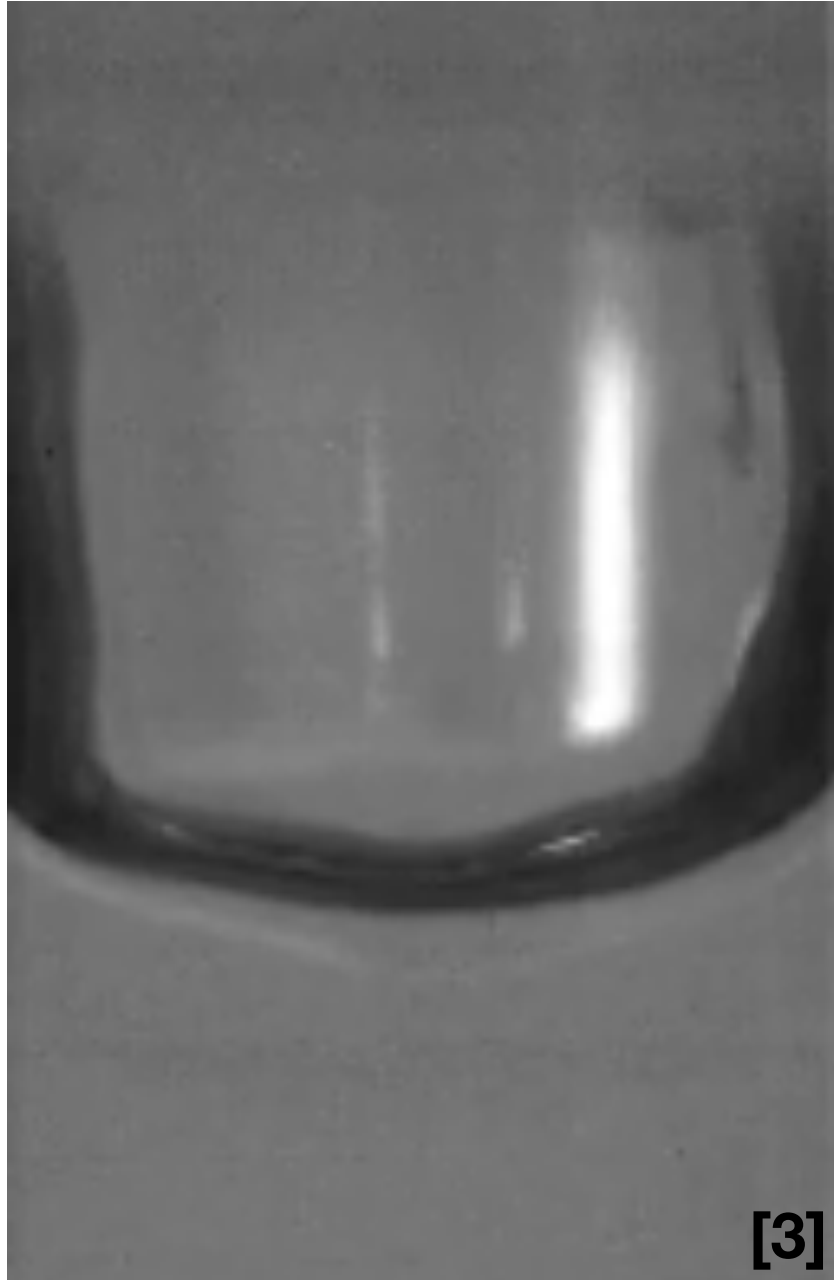


[3] Phillips, Samuel and Agarwal, Anurag and Jordan, Peter Journal, *The Sound Produced by a Dripping Tap is Driven by Resonant Oscillations of an Entrapped Air Bubble*. Scientific Reports 2018. Vol. 8 No. 1.

<https://www.nature.com/articles/s41598-018-27913-0>



Minnaert frequency





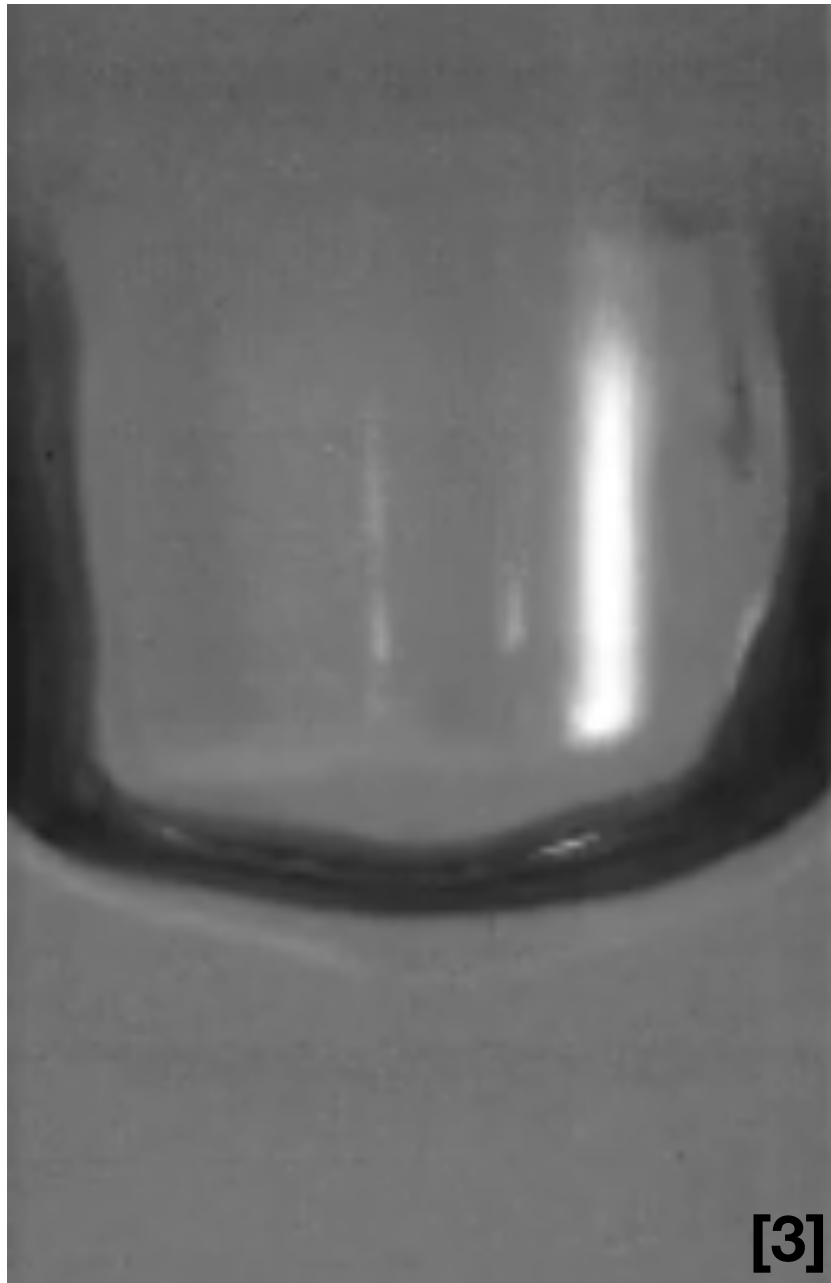
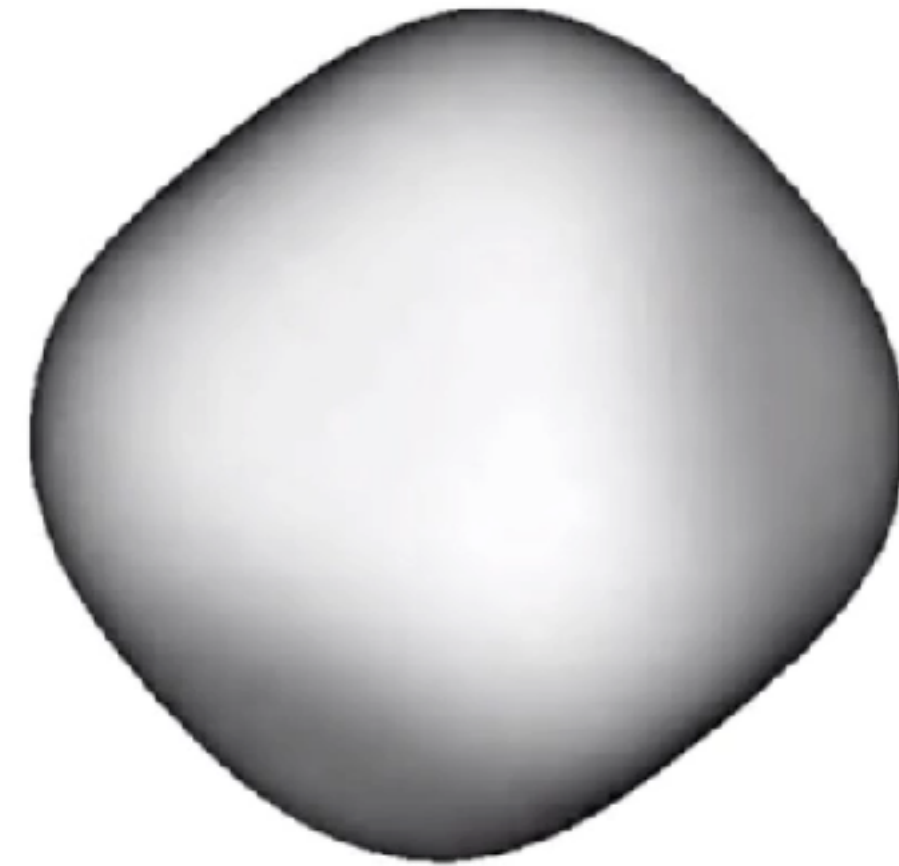
Minnaert frequency

$$f = \frac{1}{2\pi r_{bubble}} \cdot \sqrt{\frac{3\gamma p_o}{\rho_{water}}}$$

γ Adiabatic coefficient of air

p_o Outer air pressure

ρ_{water} Density of water





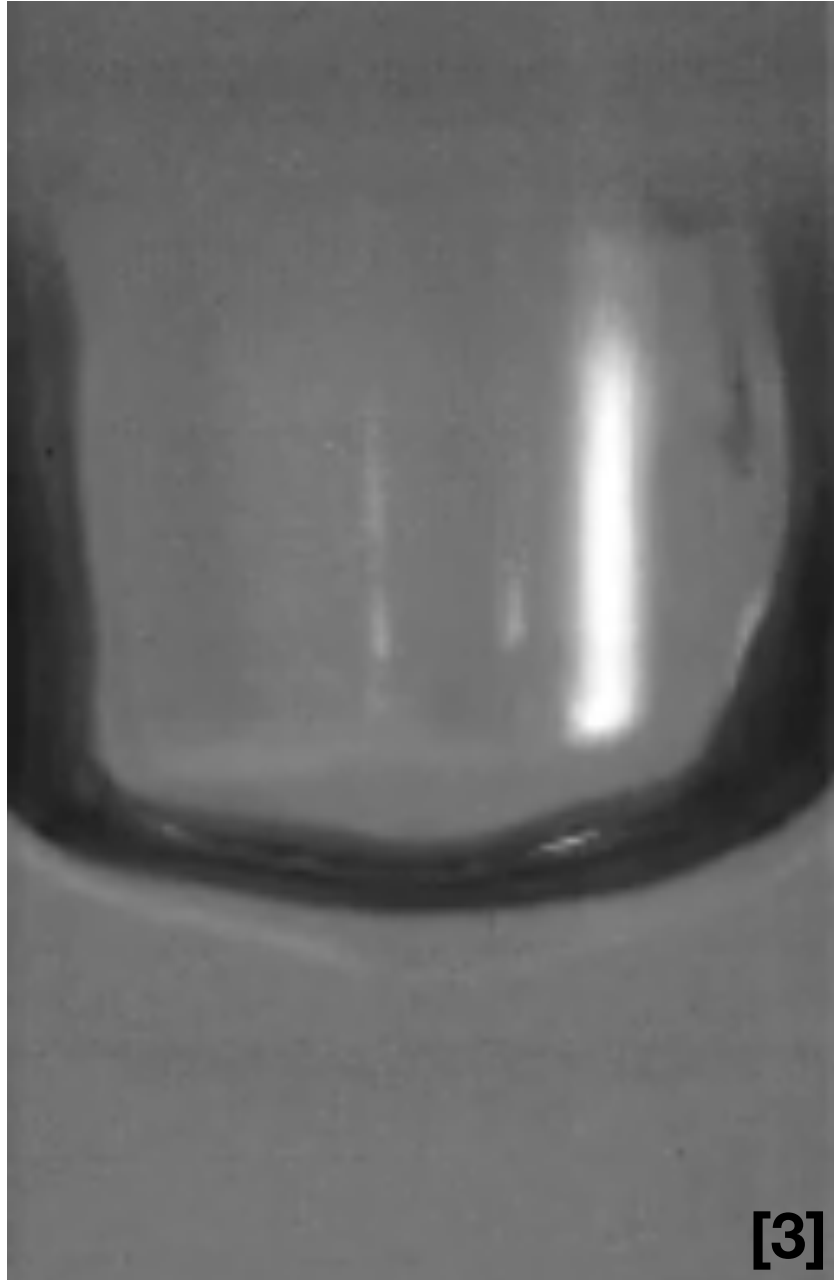
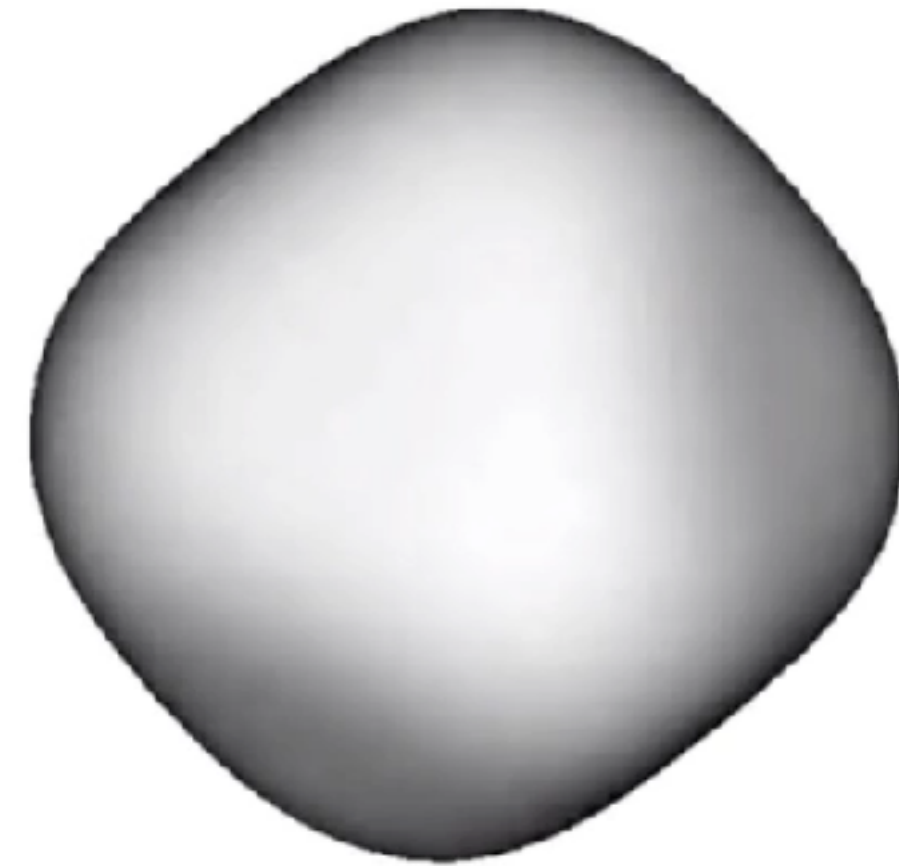
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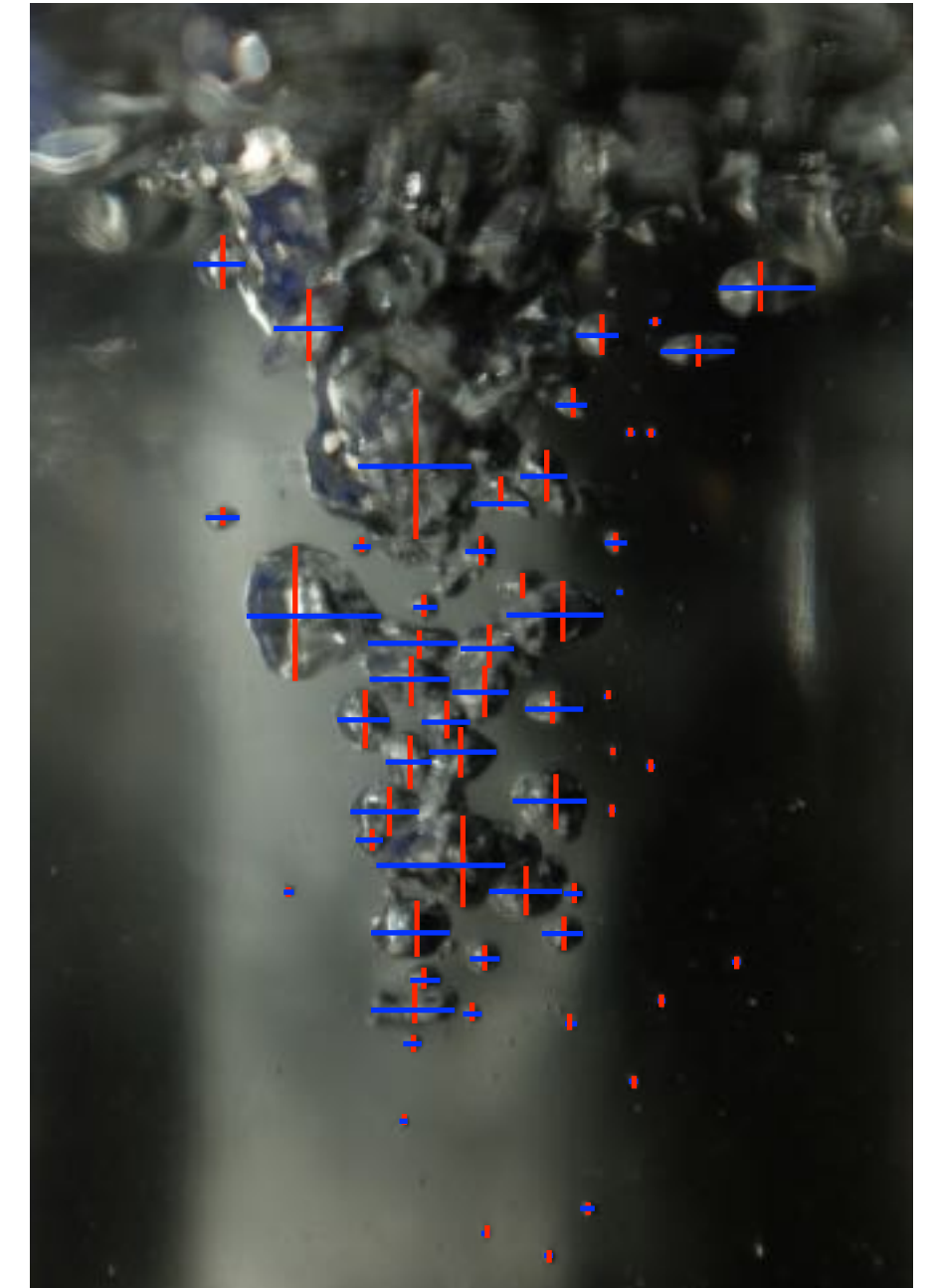
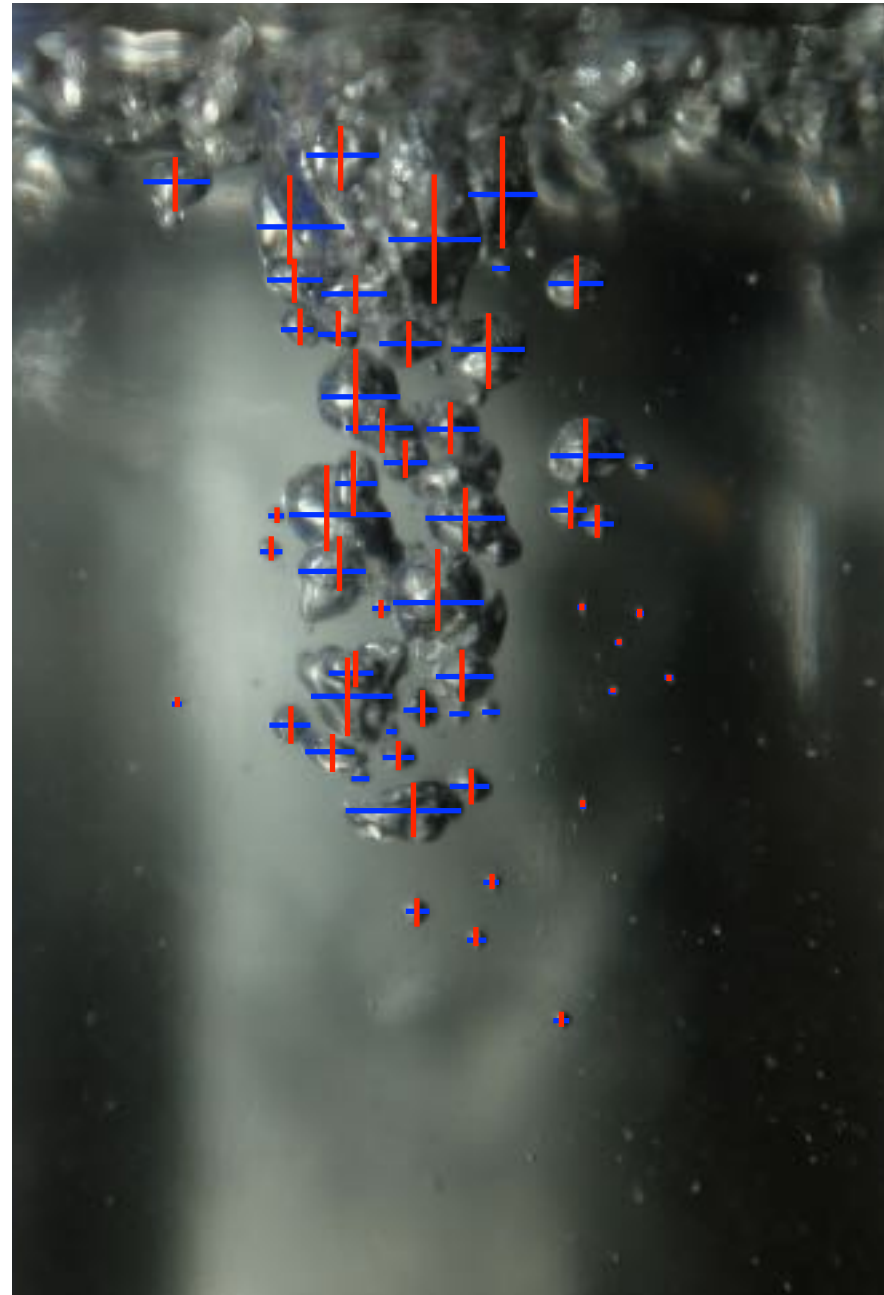
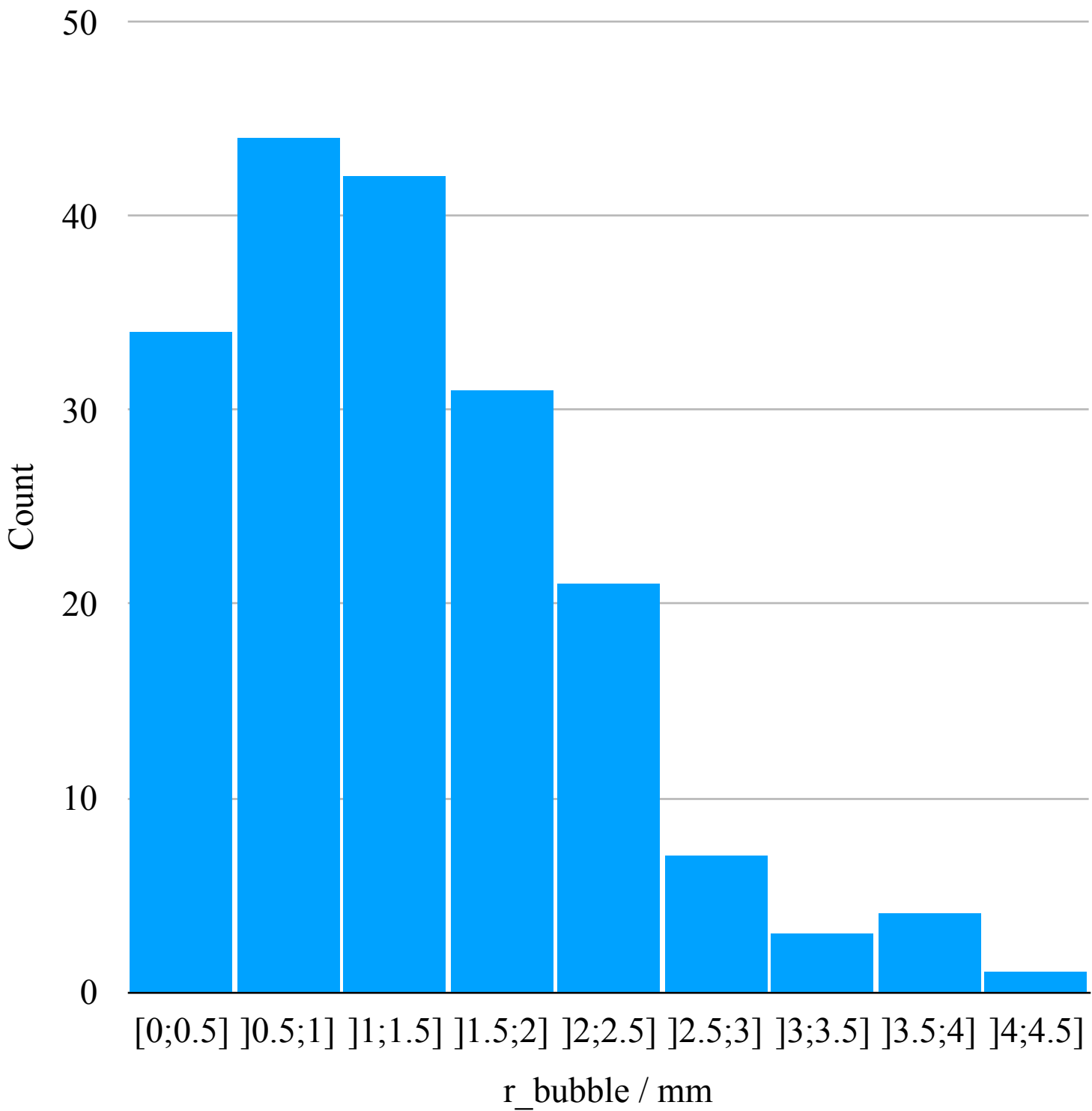


[3]

The frequencies which occur due to Minnaert resonance can be amplified by the standing waves in the cavity.



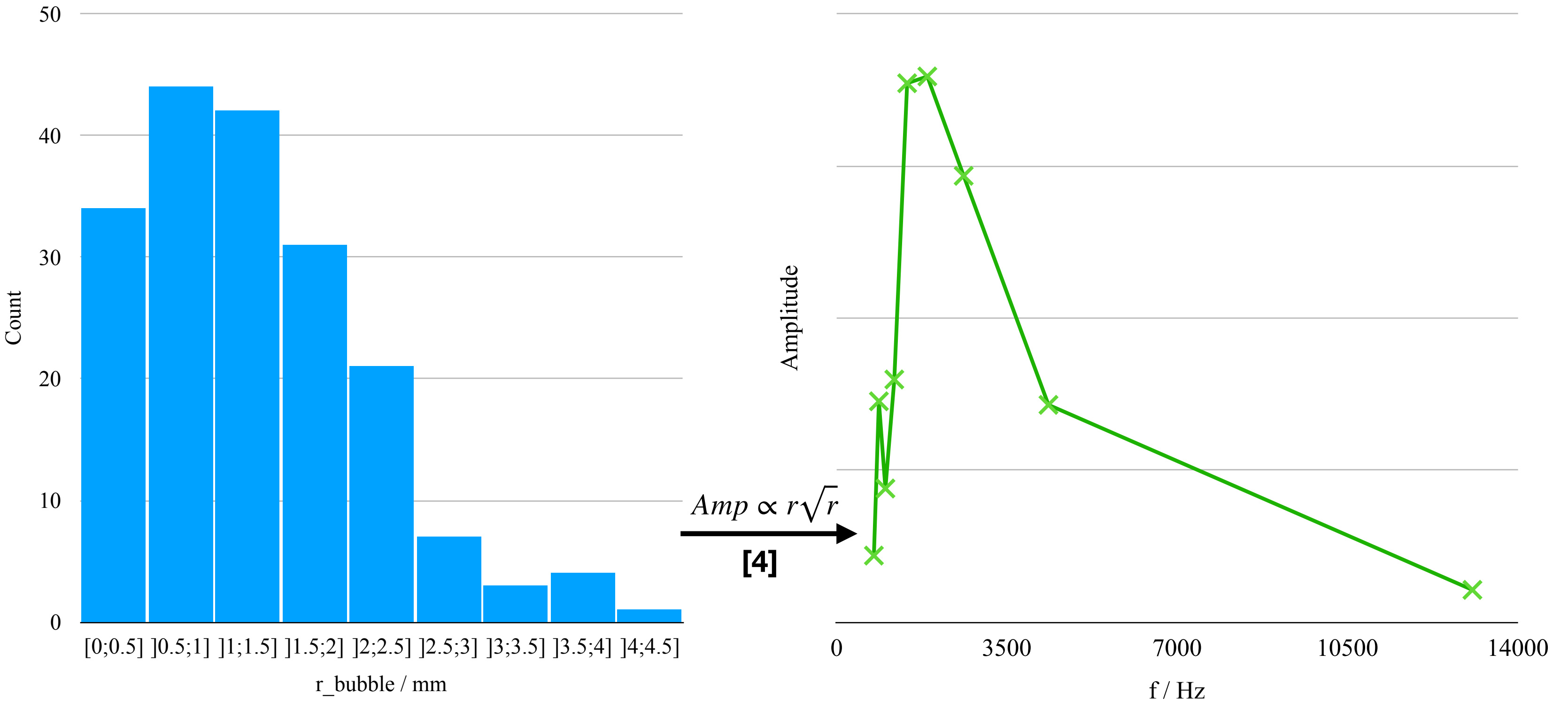
Bubble radii distribution



Pictures evaluated

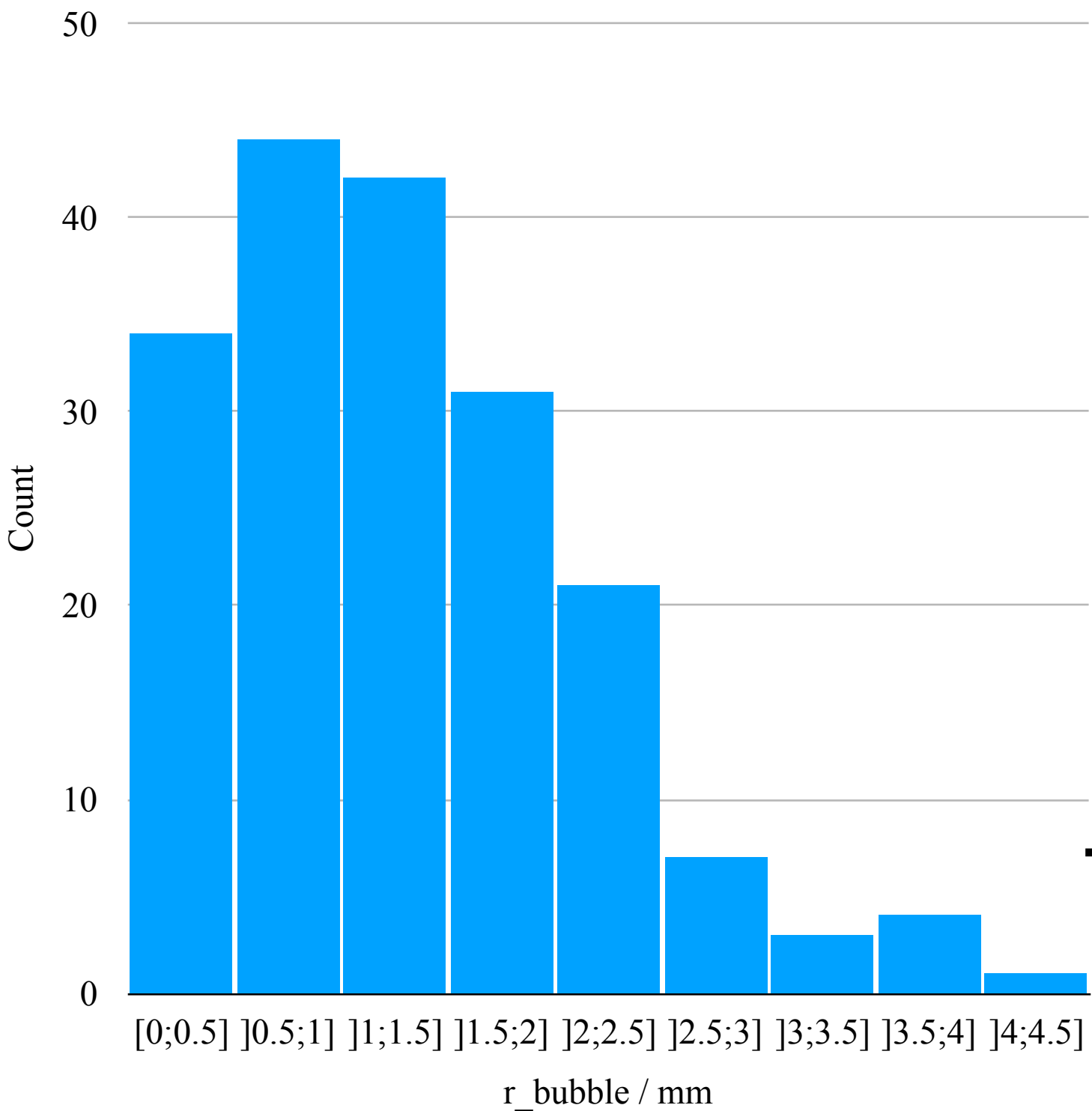


Spectrum created (qualitative)

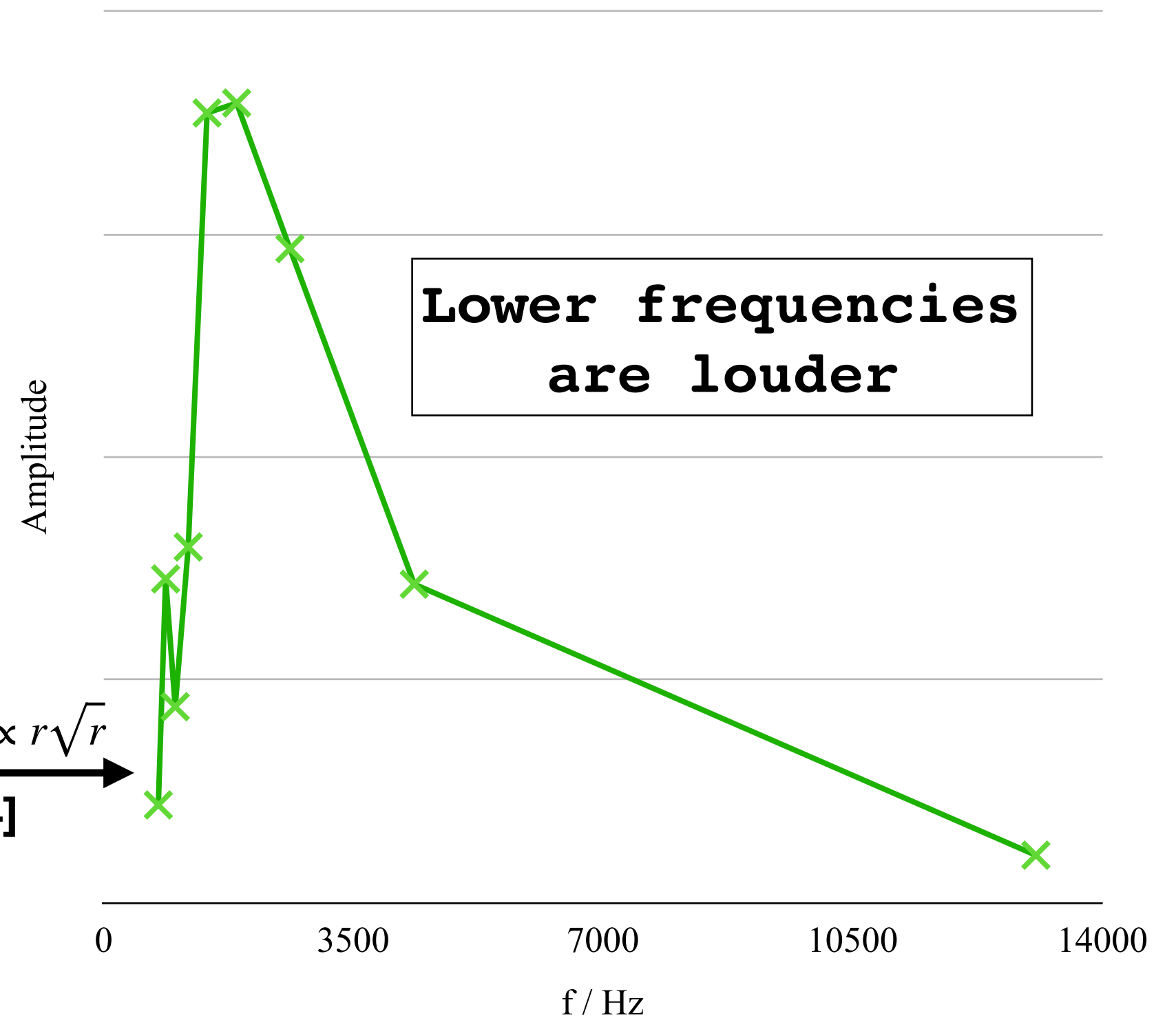




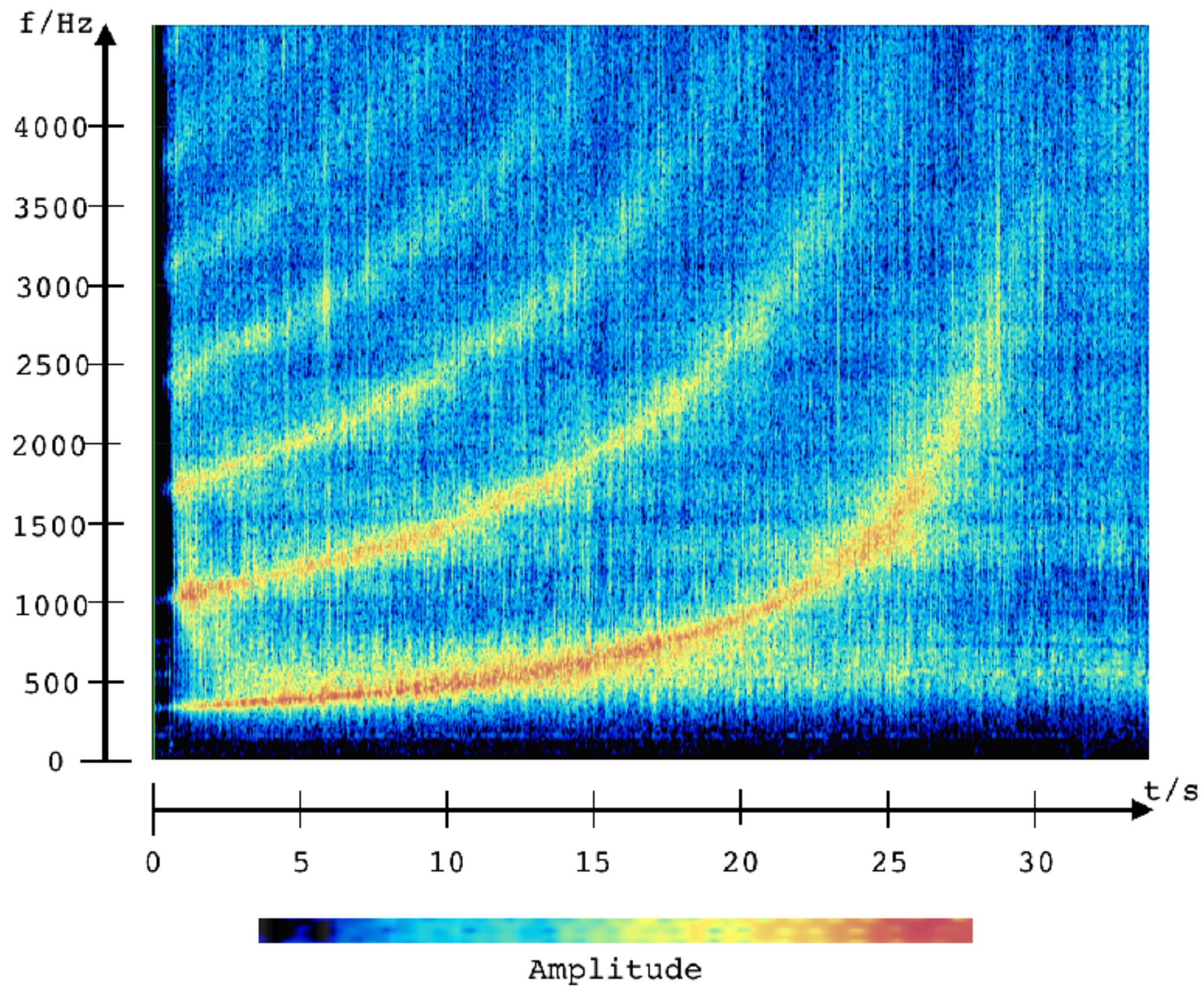
Spectrum created (qualitative)

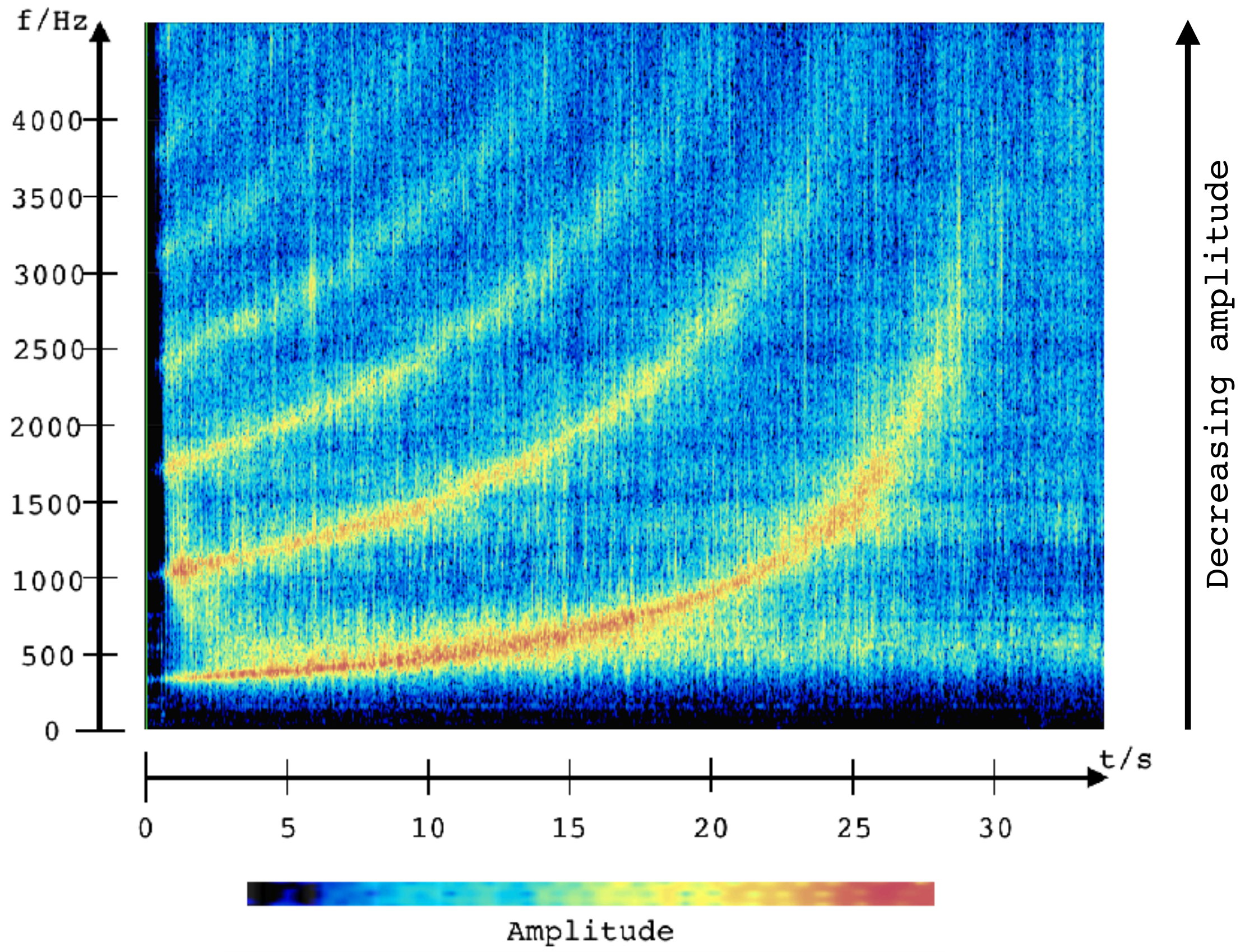


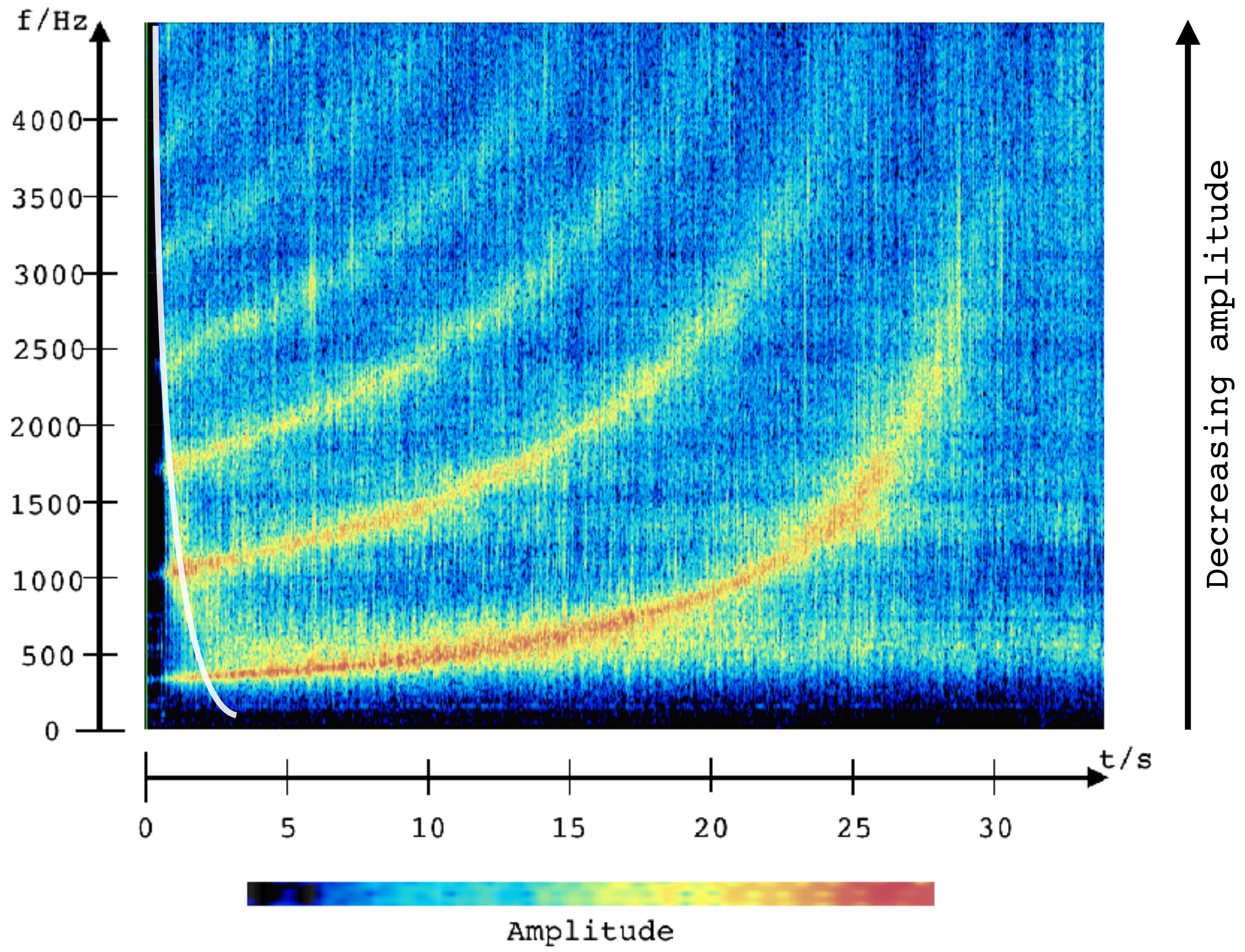
$Amp \propto r\sqrt{r}$
[4]



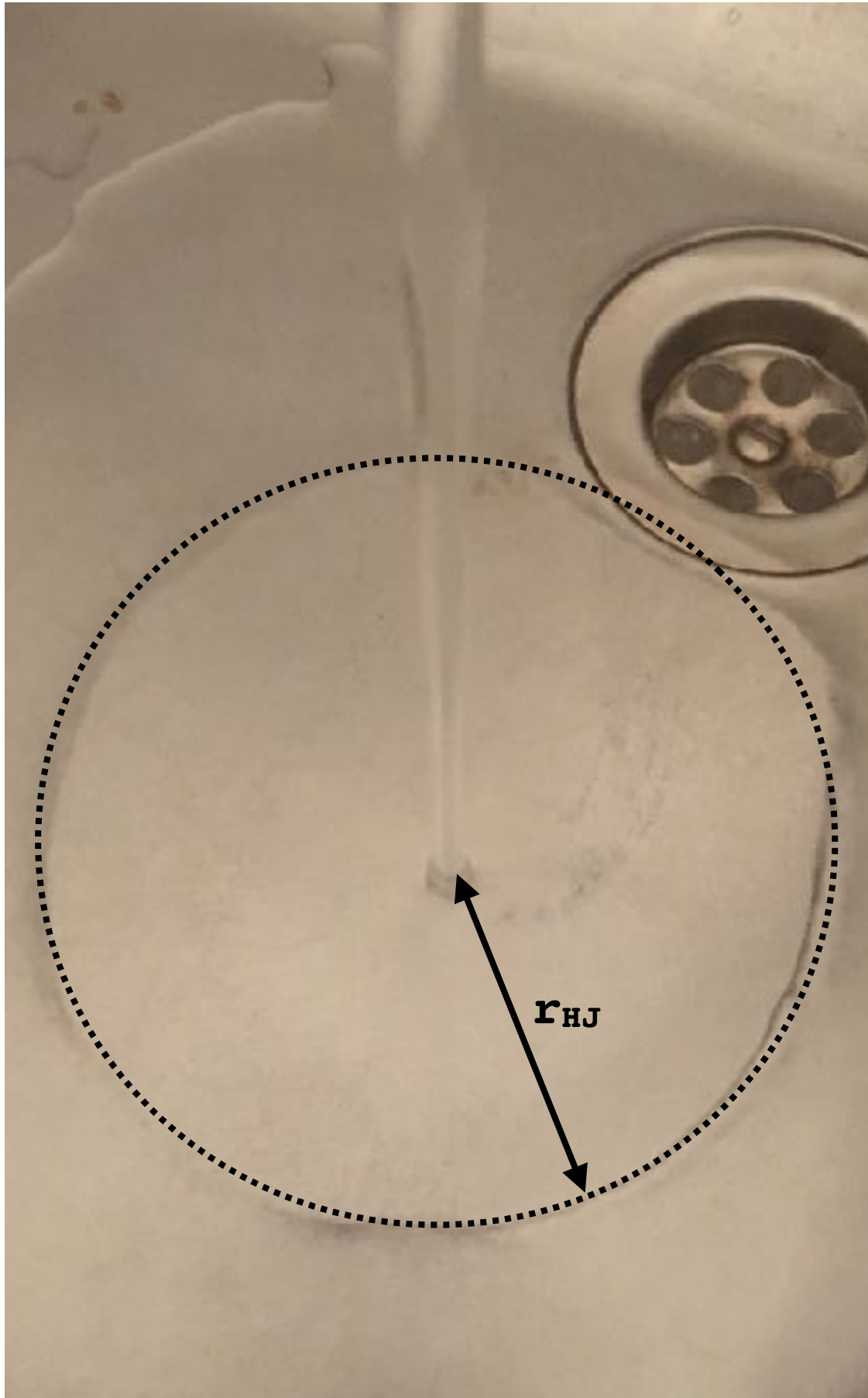
[4] Doel, Kees van den. "Physically based models for liquid sounds." (2004)

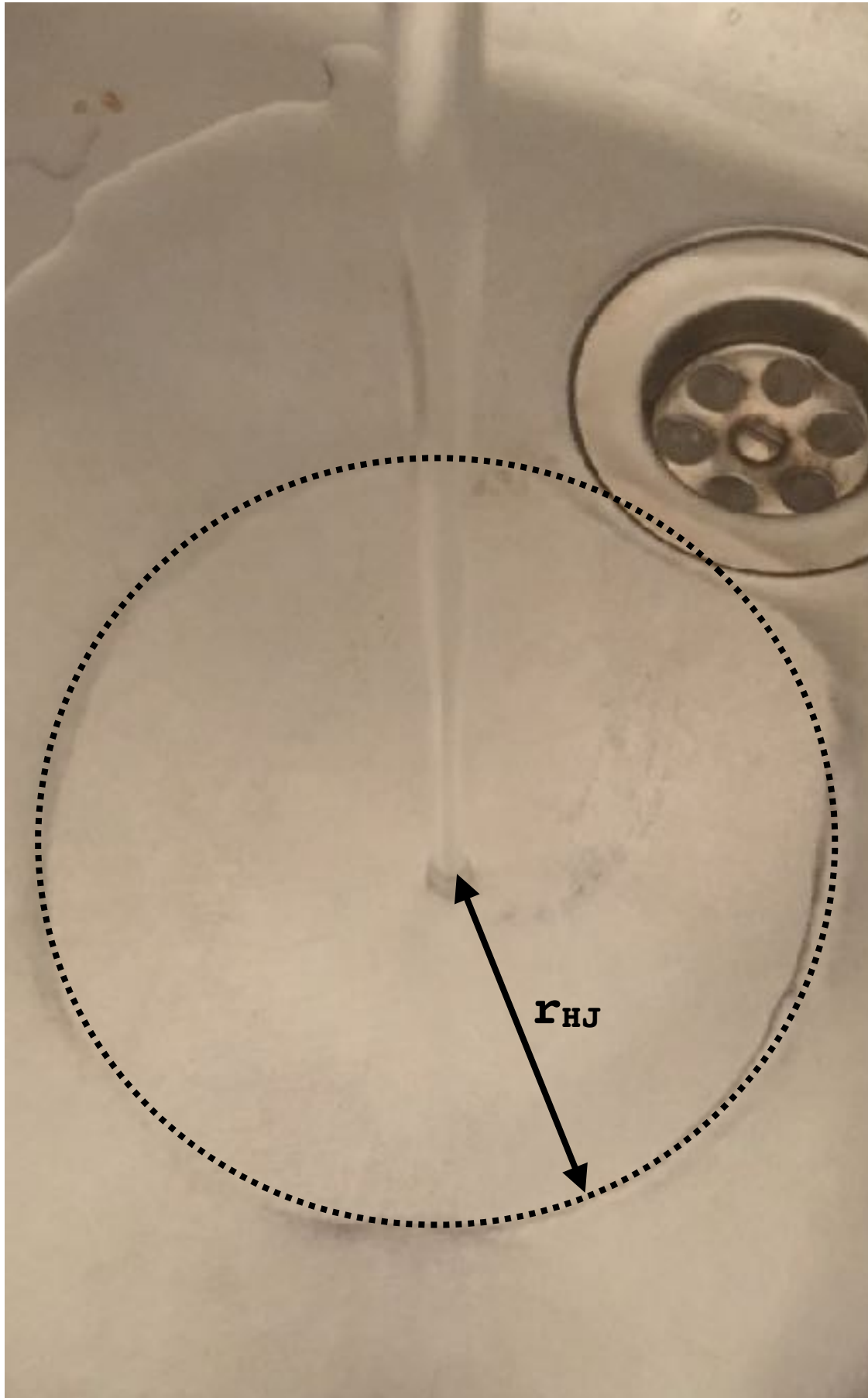


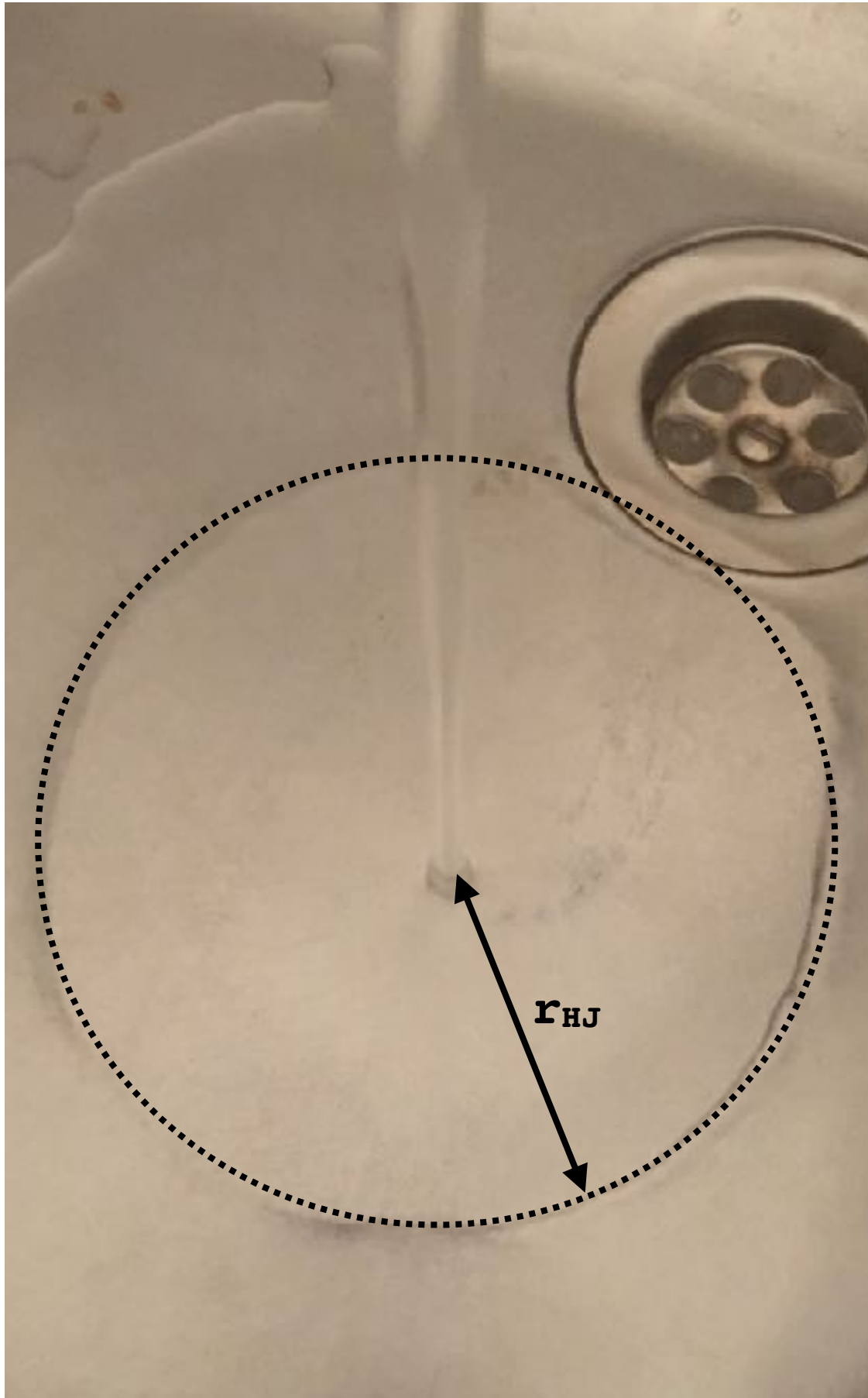










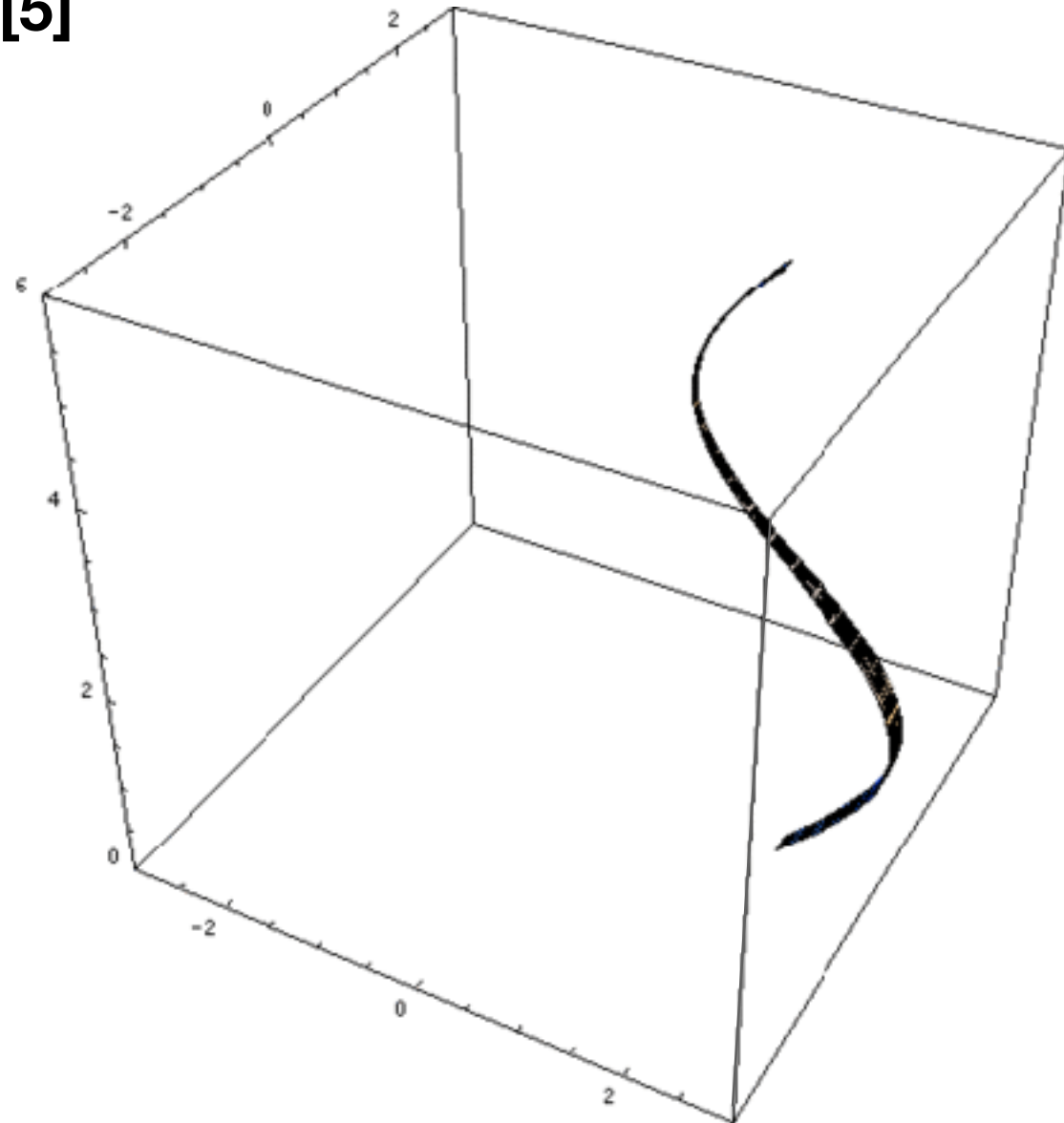


16 r_{HJ} decreases with increasing h_w



Modeling all possible bottles: Change of h_c

[5]



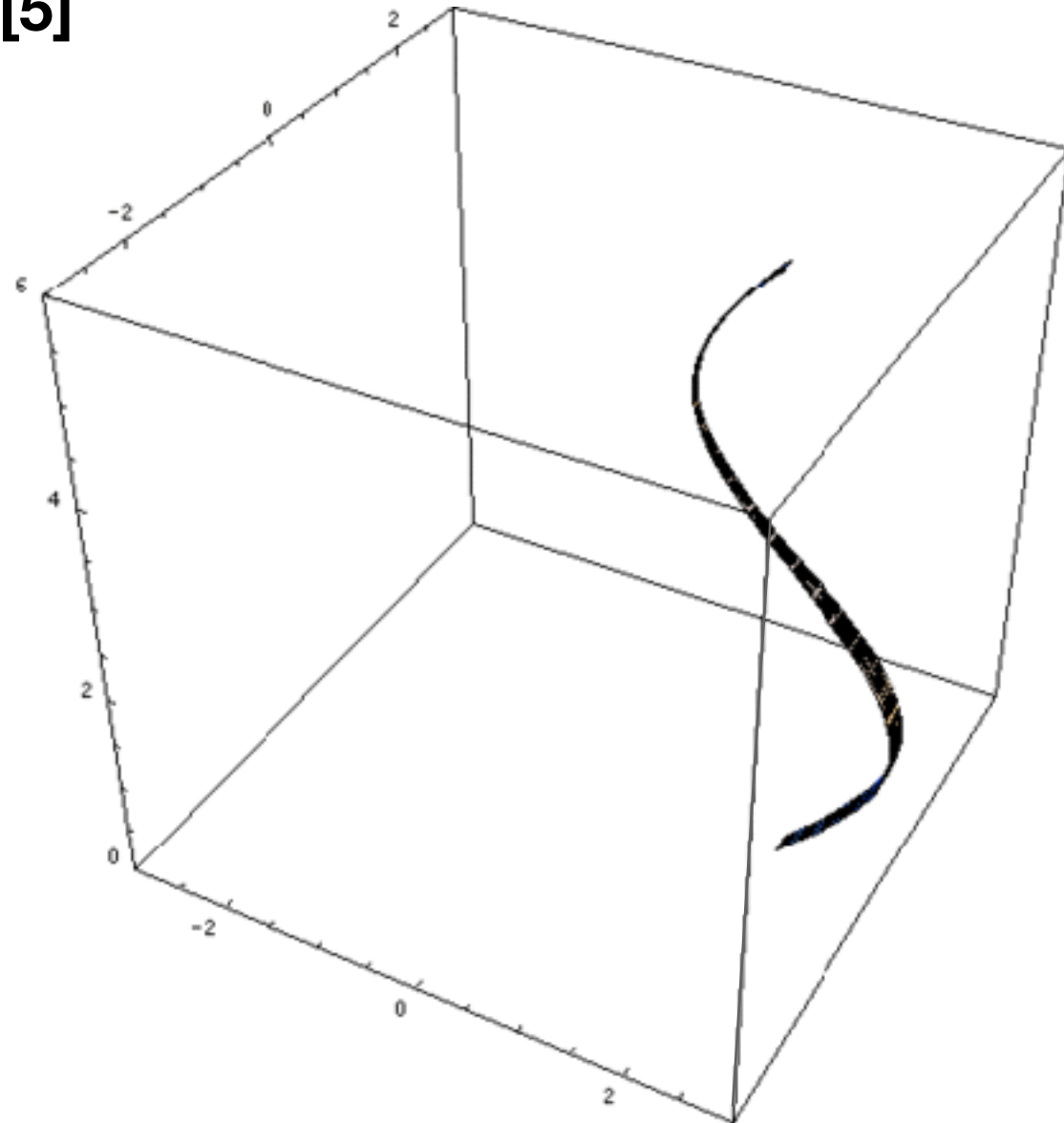
[5]

https://commons.wikimedia.org/wiki/File:Rotationskoerper_animation.gif



Modeling all possible bottles: Change of h_c

[5]



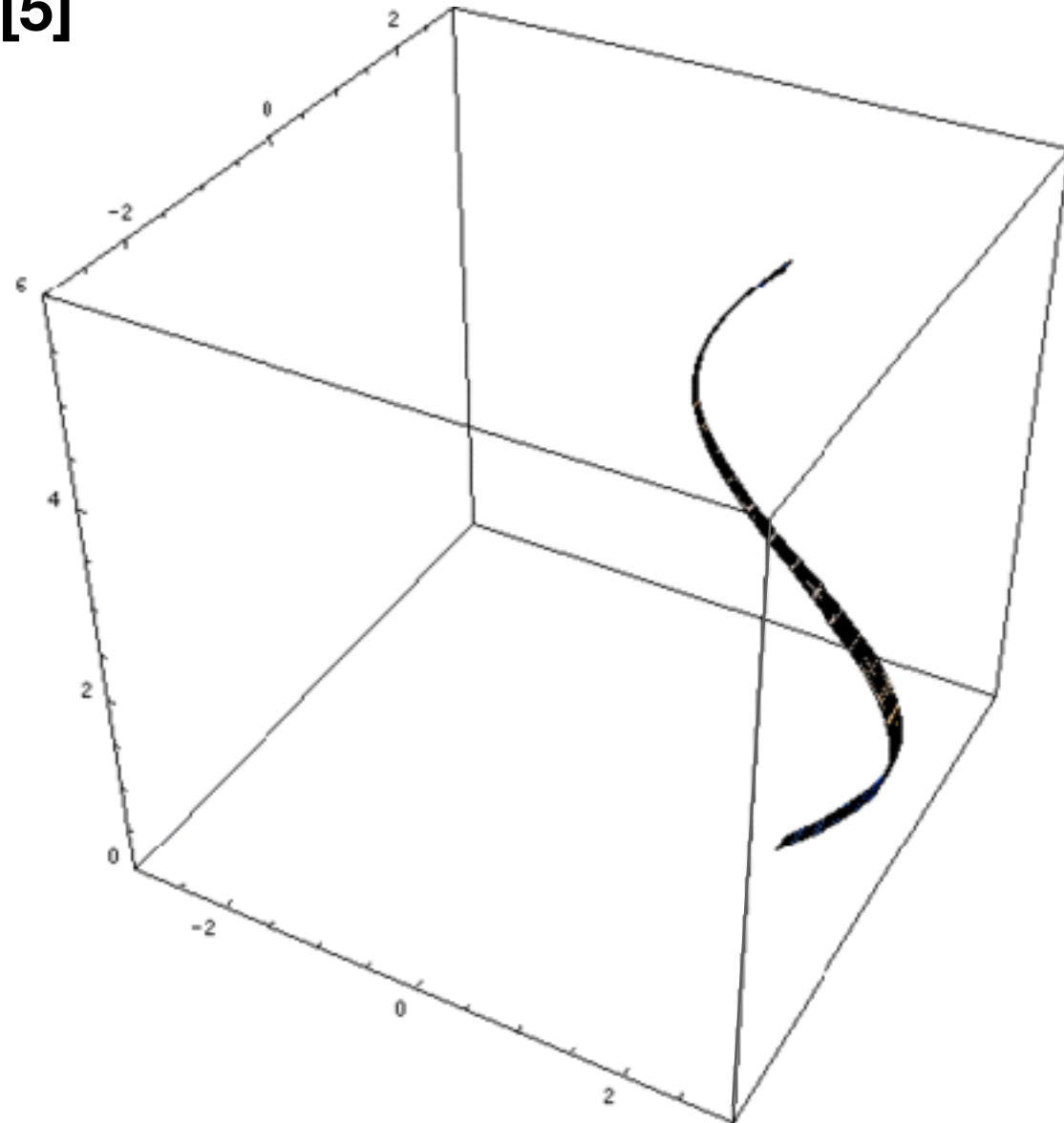
[5]

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Modeling all possible bottles: Change of h_c

[5]



The changing rate of the water height in the container depends on the cross-sectional area at this height.

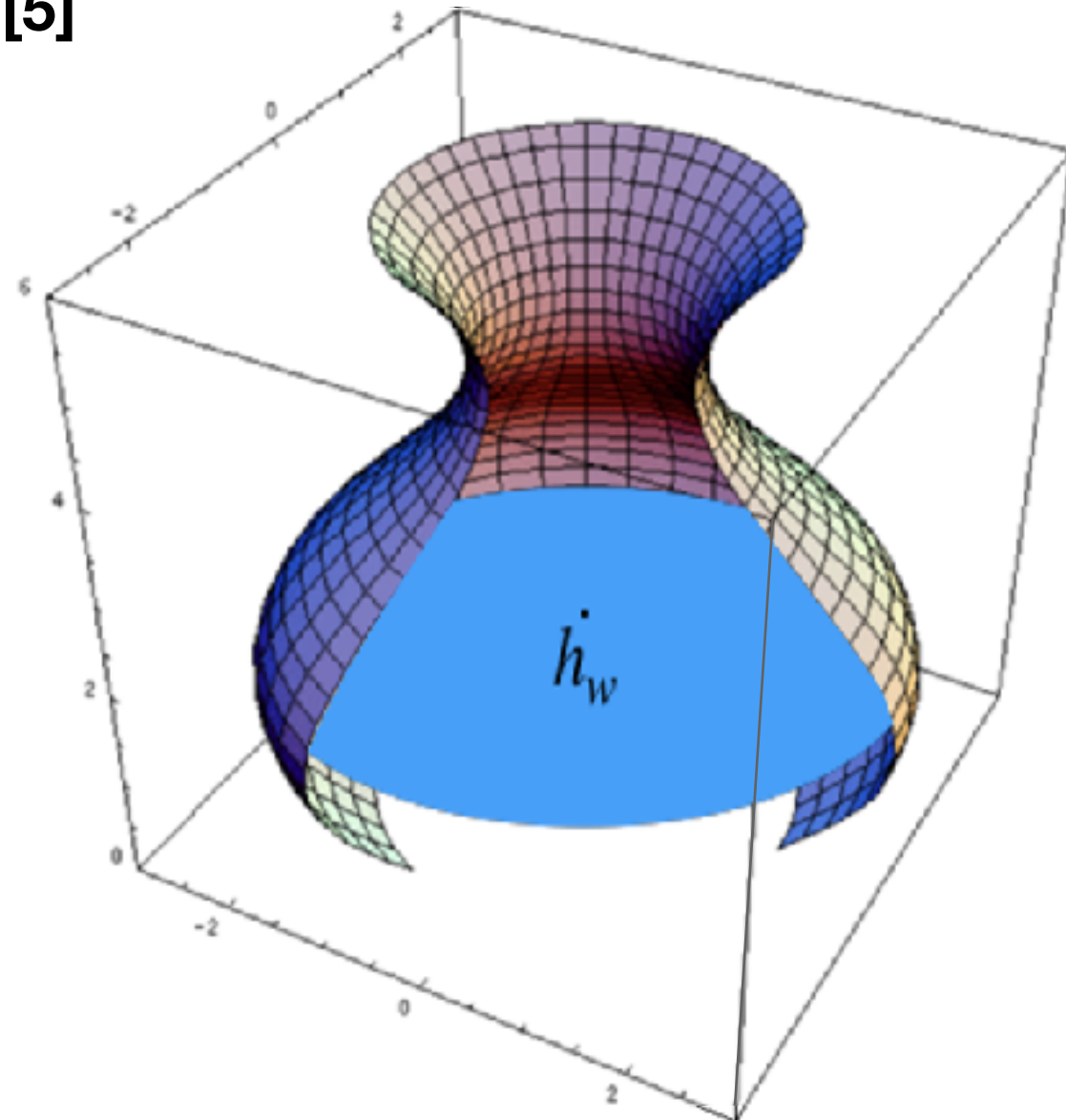
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https://commons.wikimedia.org/wiki/File:Rotationskoerper_animation.gif



Modeling all possible bottles: Change of h_c

[5]



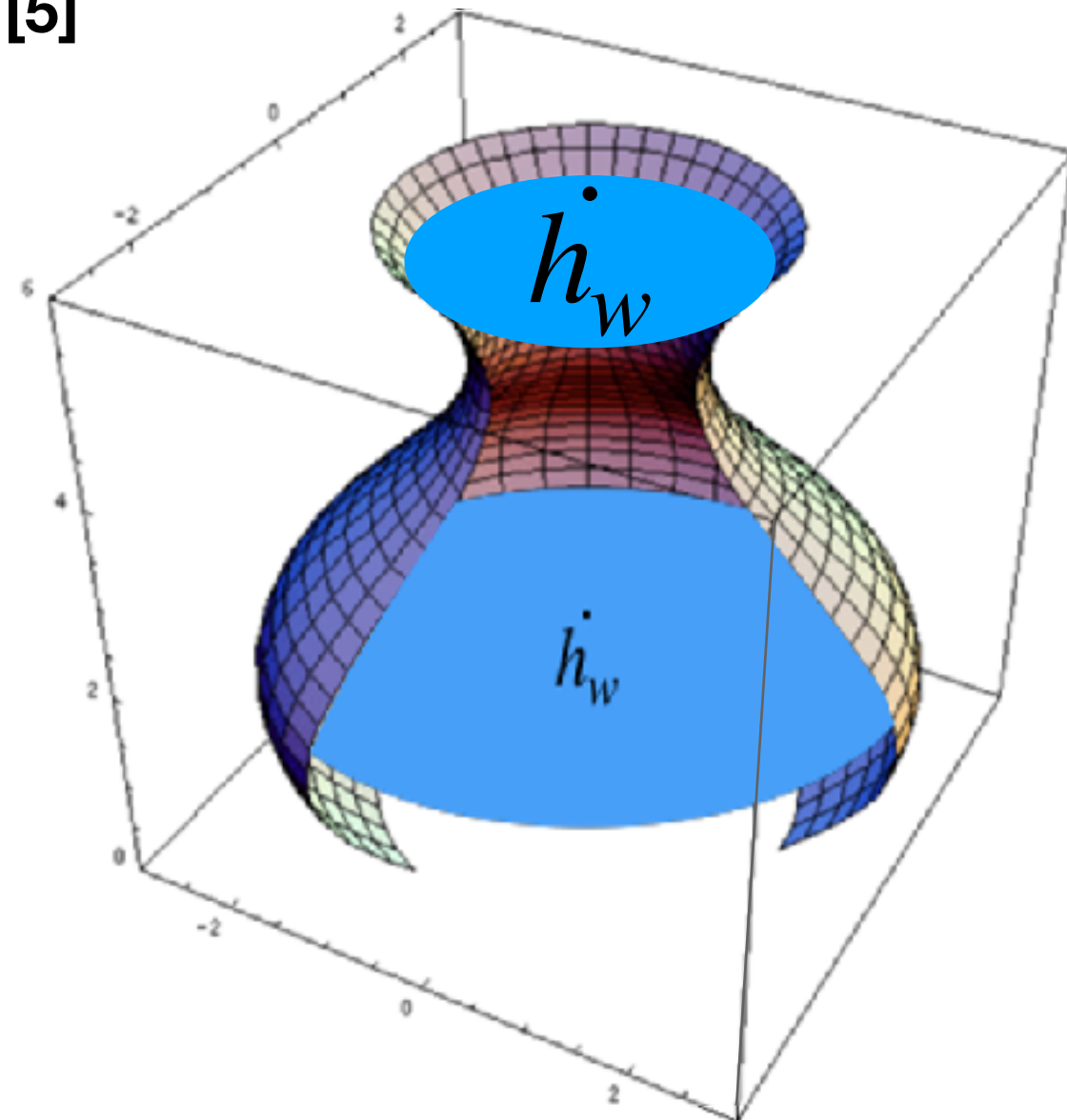
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Modeling all possible bottles: Change of h_c

[5]



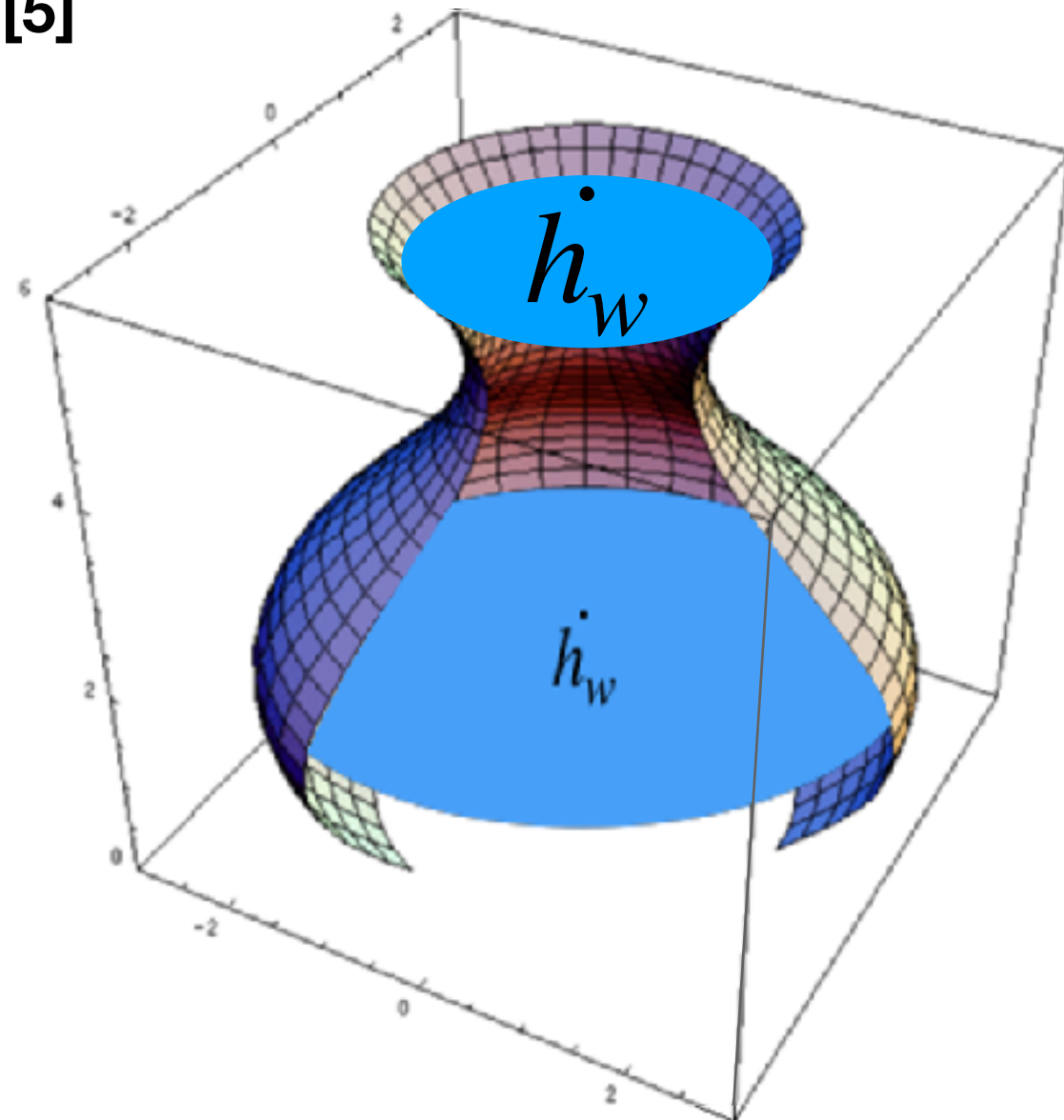
The changing rate of the water height in the container depends on the cross-sectional area at this height.

[5] https://commons.wikimedia.org/wiki/File:Rotationskoerper_animation.gif



Modeling all possible bottles: Change of h_c

[5]



The changing rate of the water height in the container depends on the cross-sectional area at this height.

$$h_c(t) = h_B - h_w(t)$$

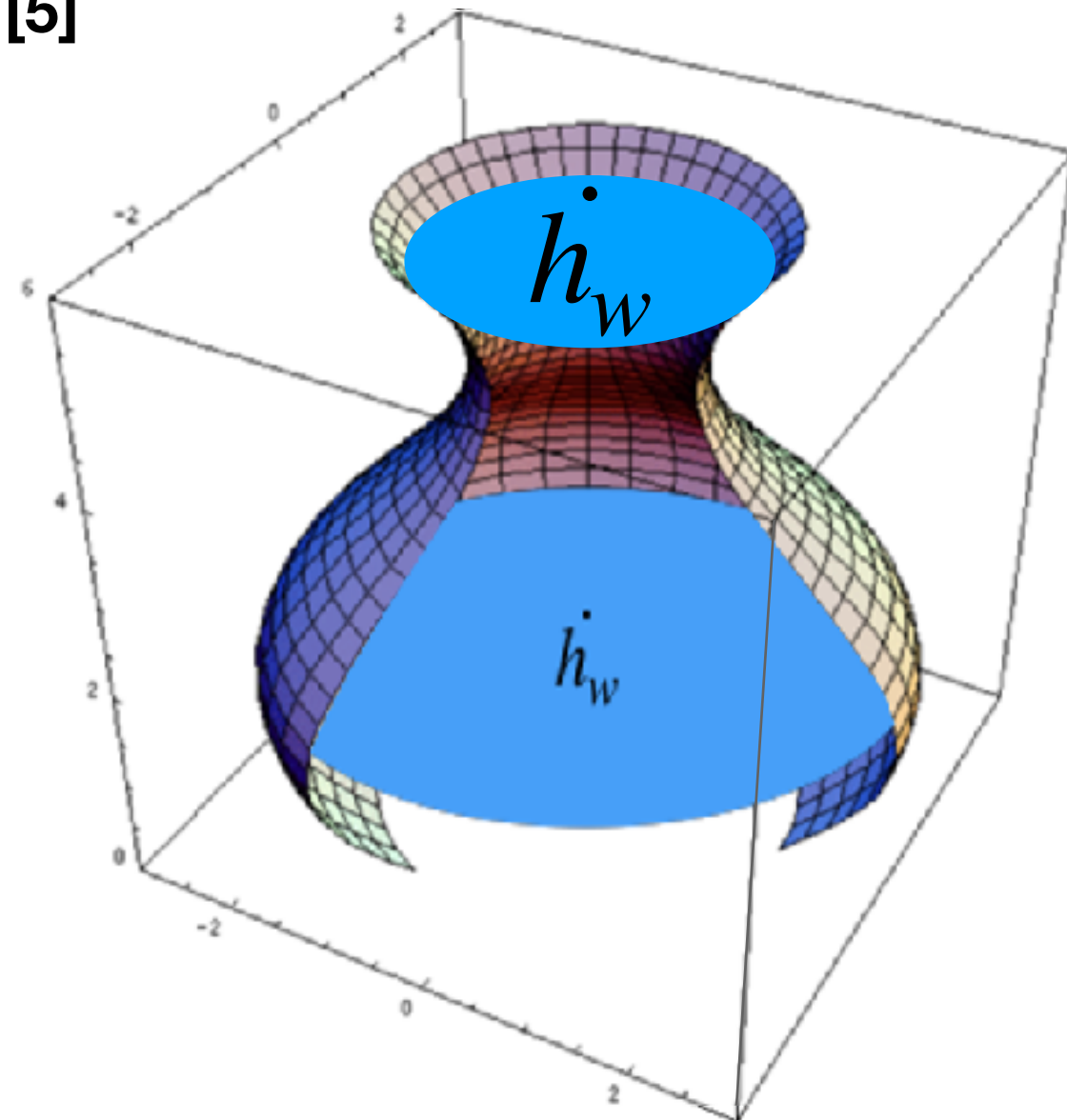
[5]

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Modeling all possible bottles: Change of h_c

[5]



The changing rate of the water height in the container depends on the cross-sectional area at this height.

$$h_c(t) = h_B - h_w(t)$$

The height of the cavity can be calculated for every container for every given moment in time.

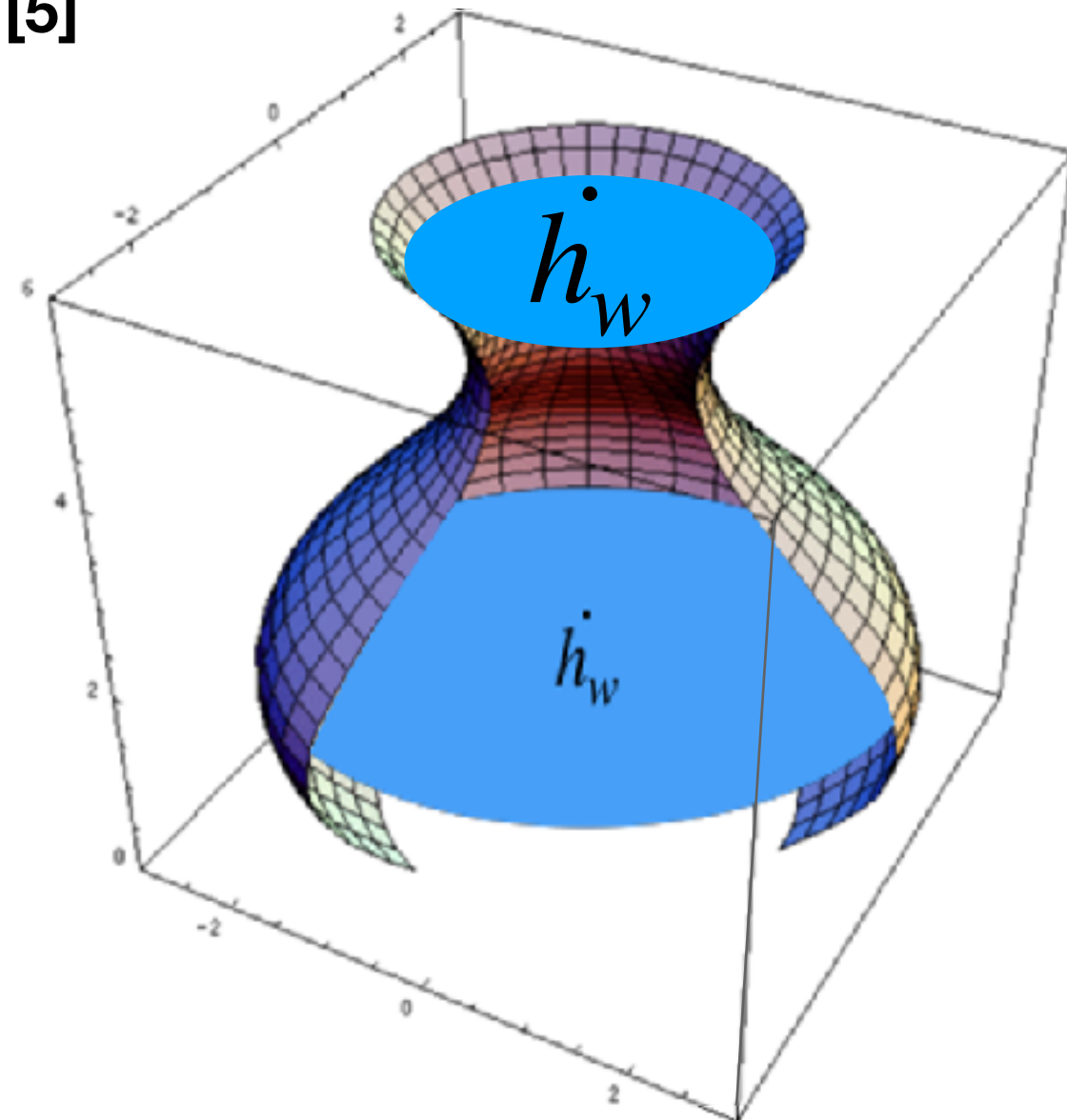
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Modeling all possible bottles: Change of h_c

[5]



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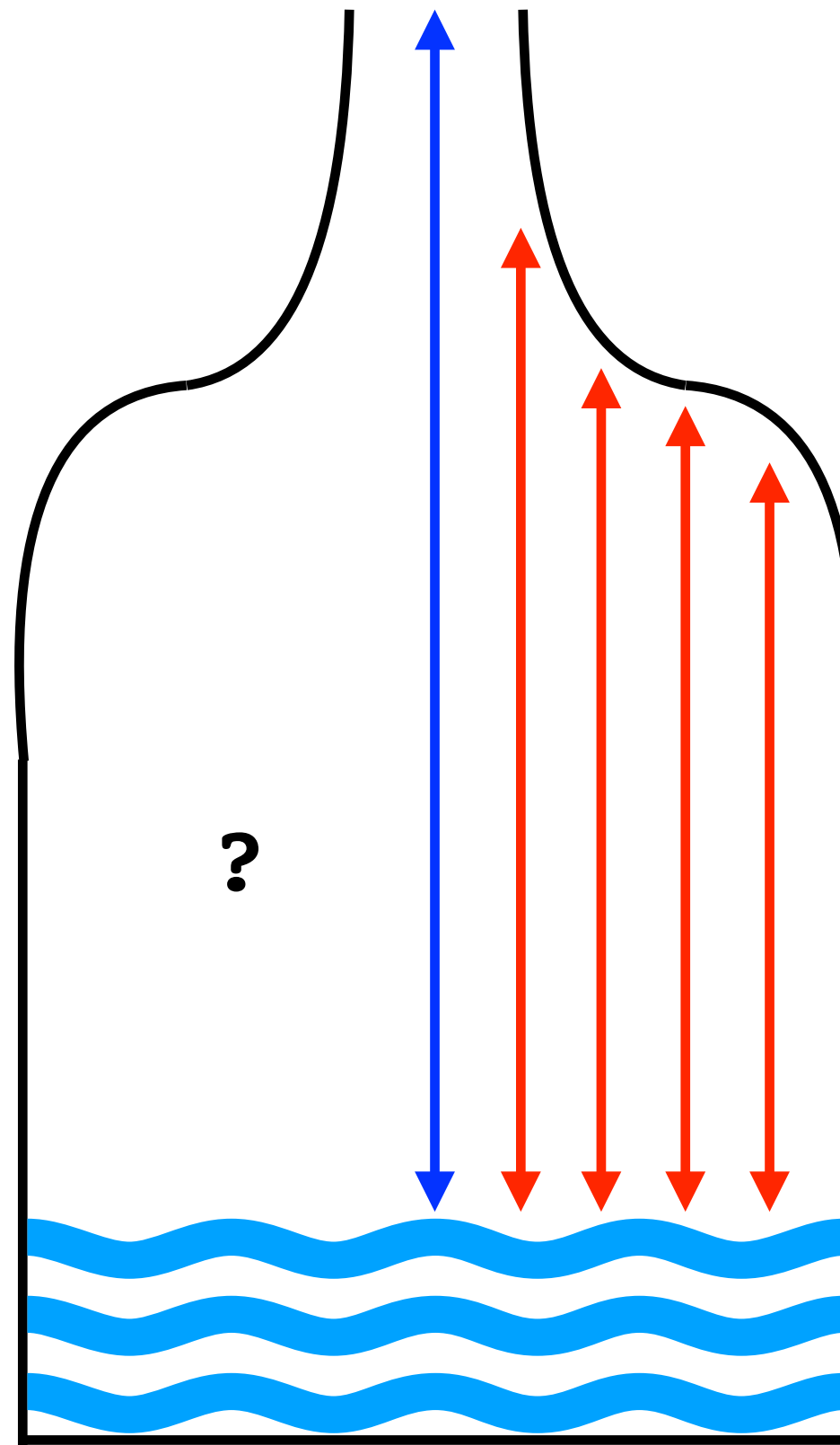
$$h_c(t) = h_B - h_w(t)$$

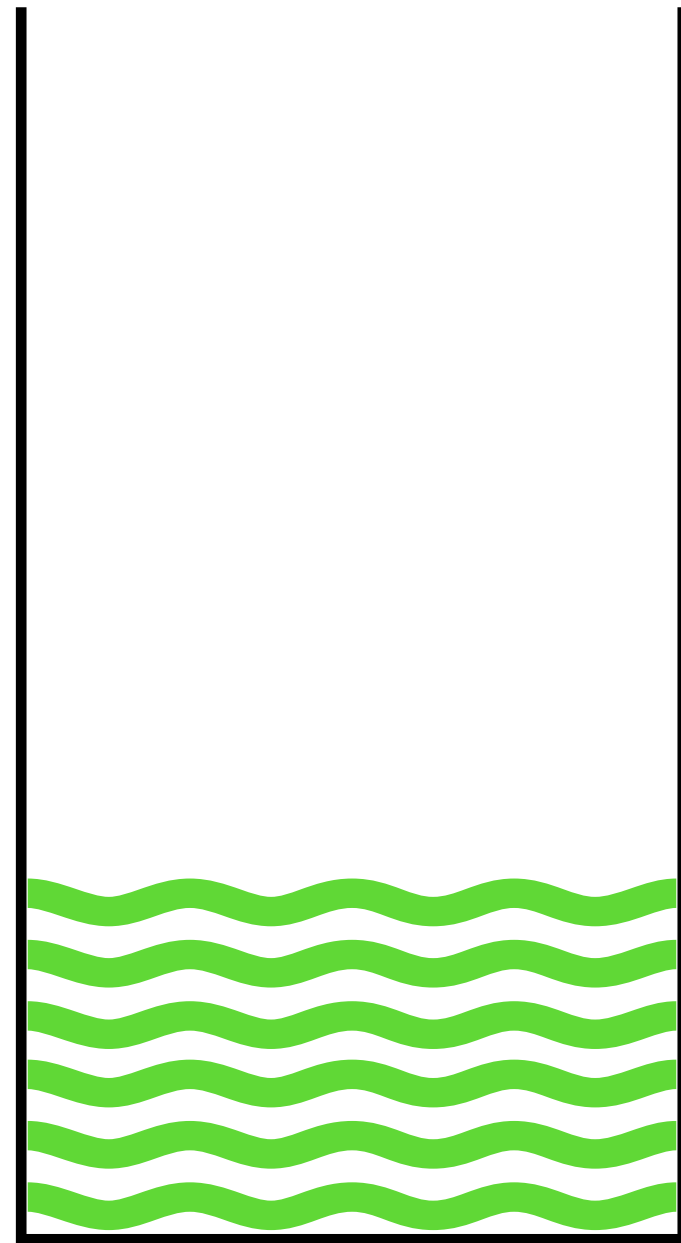
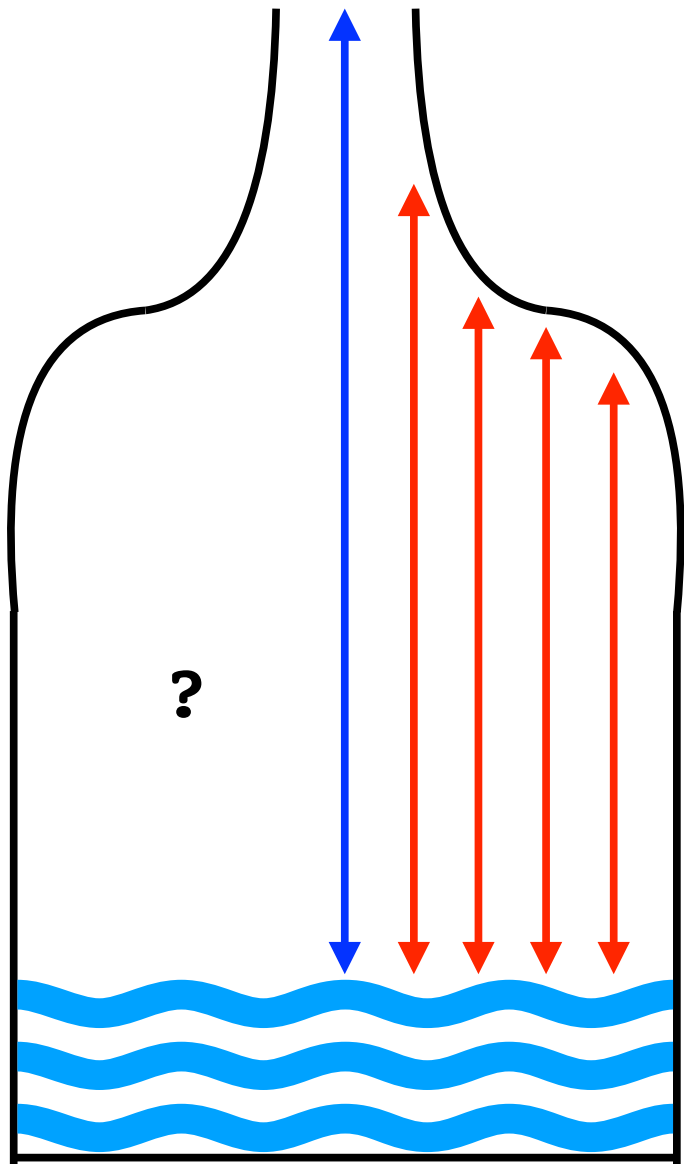
The height of the cavity can be calculated for every container for every given moment in time.

$$f_0 = \frac{c}{4h_c(t)}$$

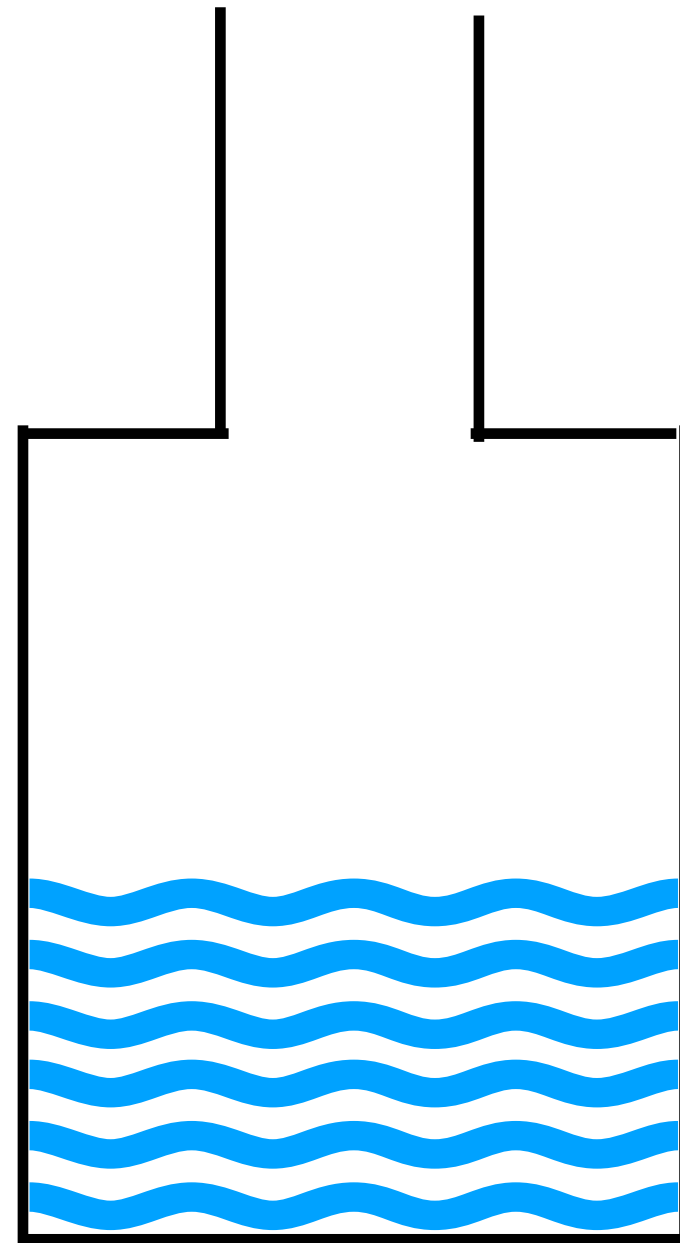
[5]

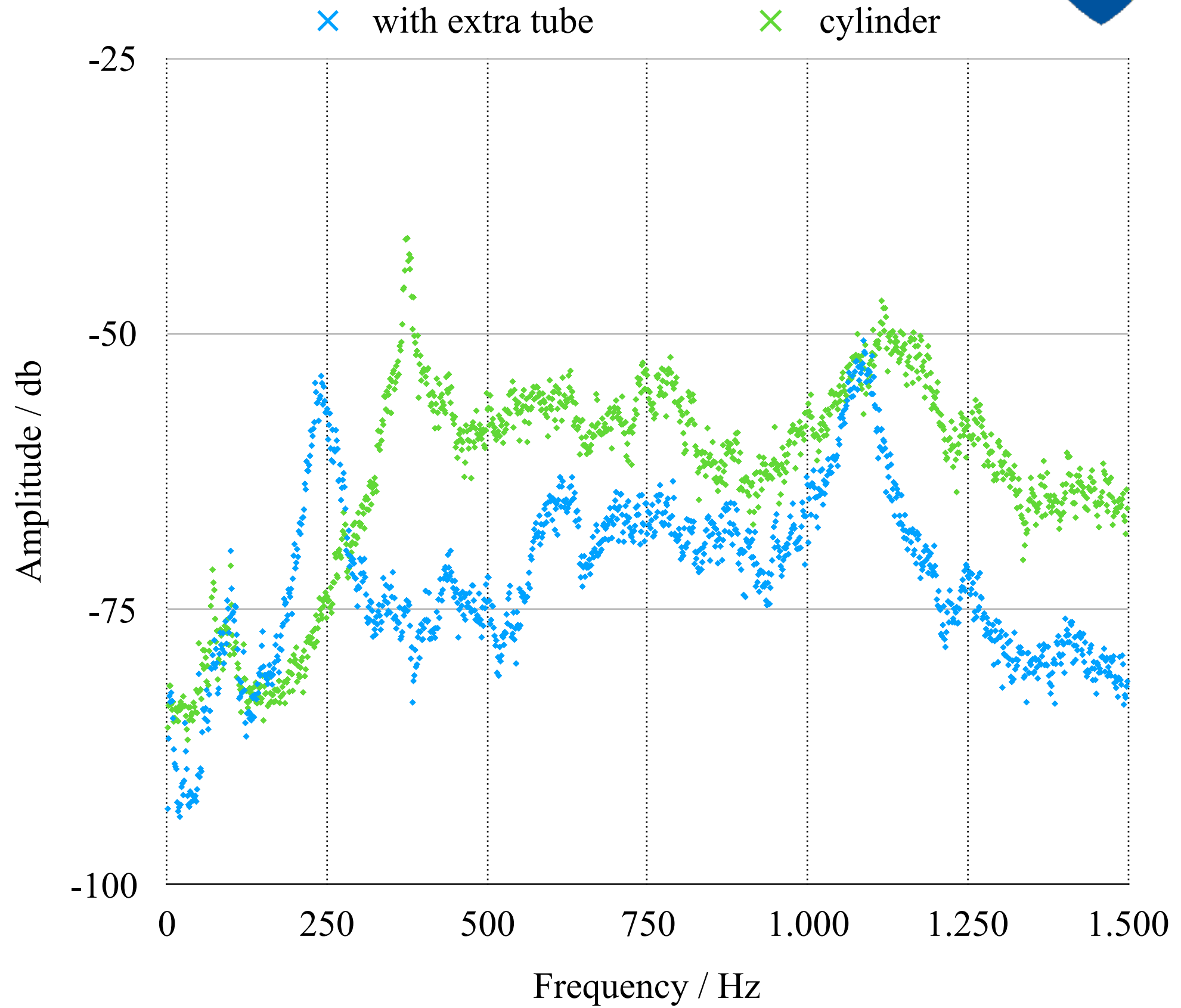
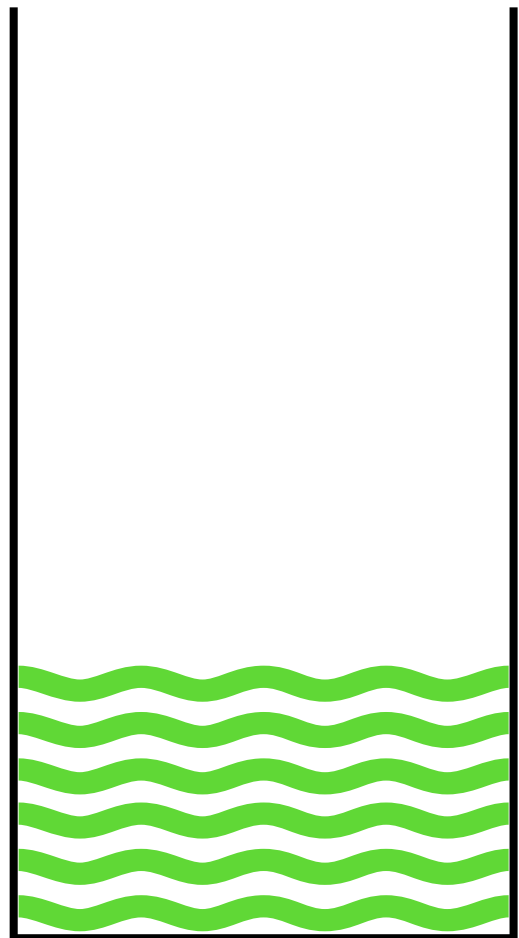
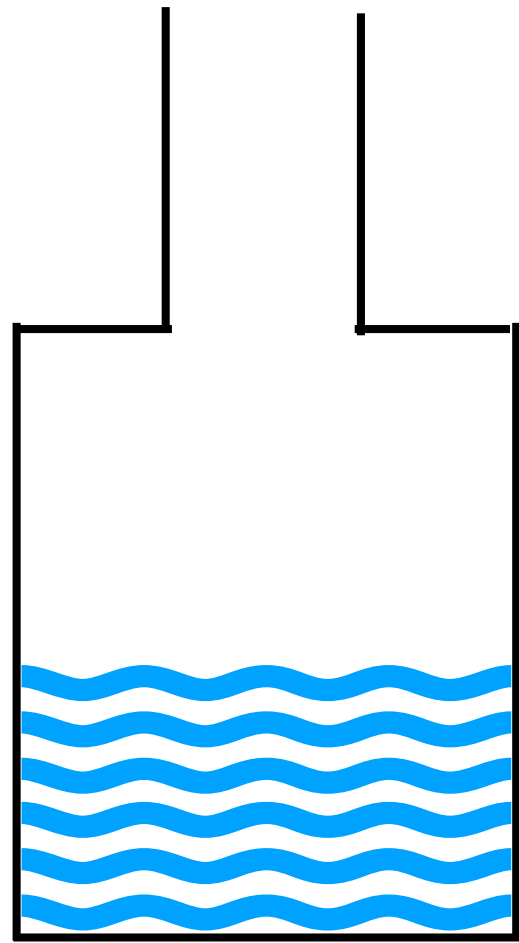
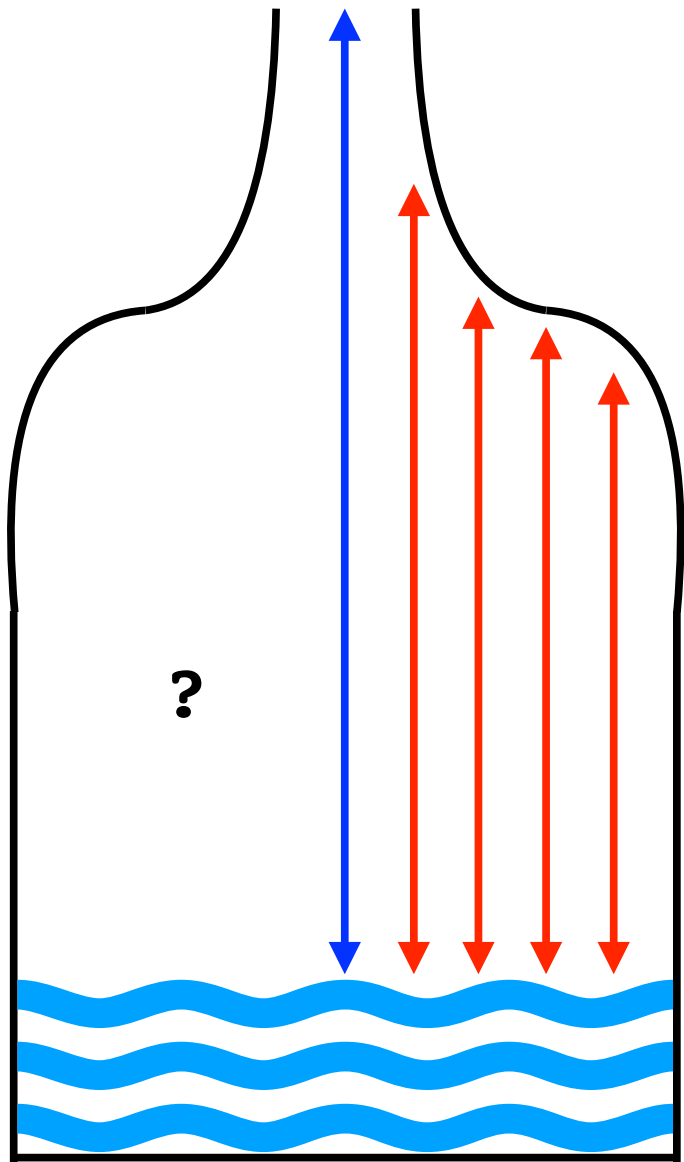
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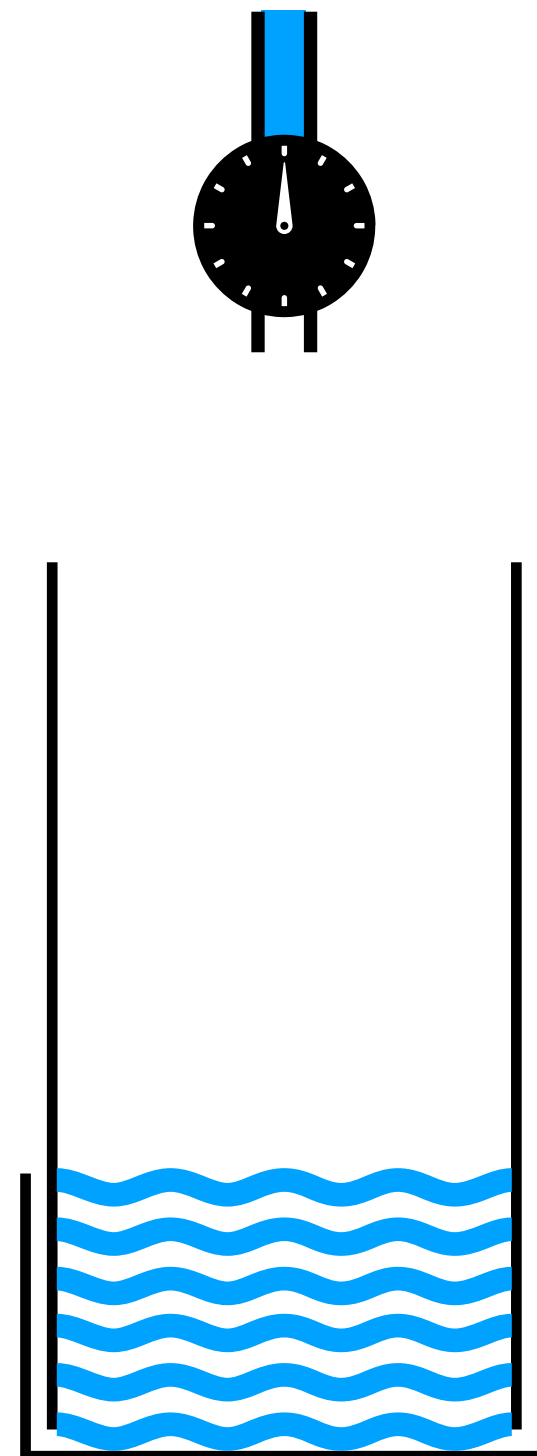
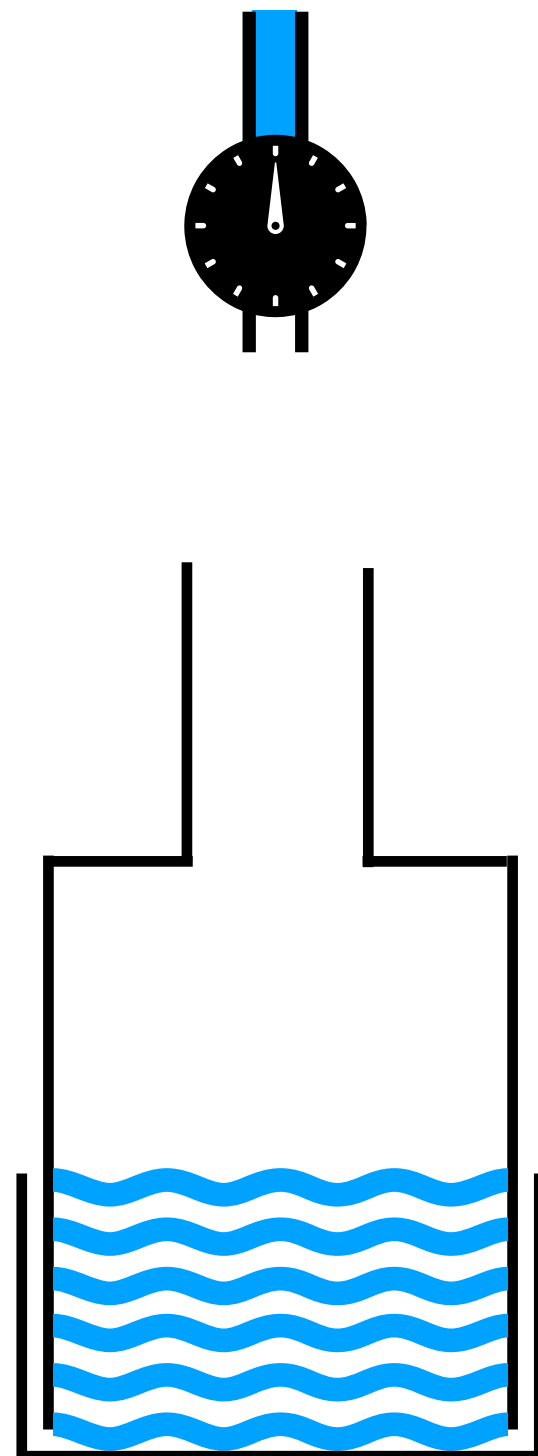


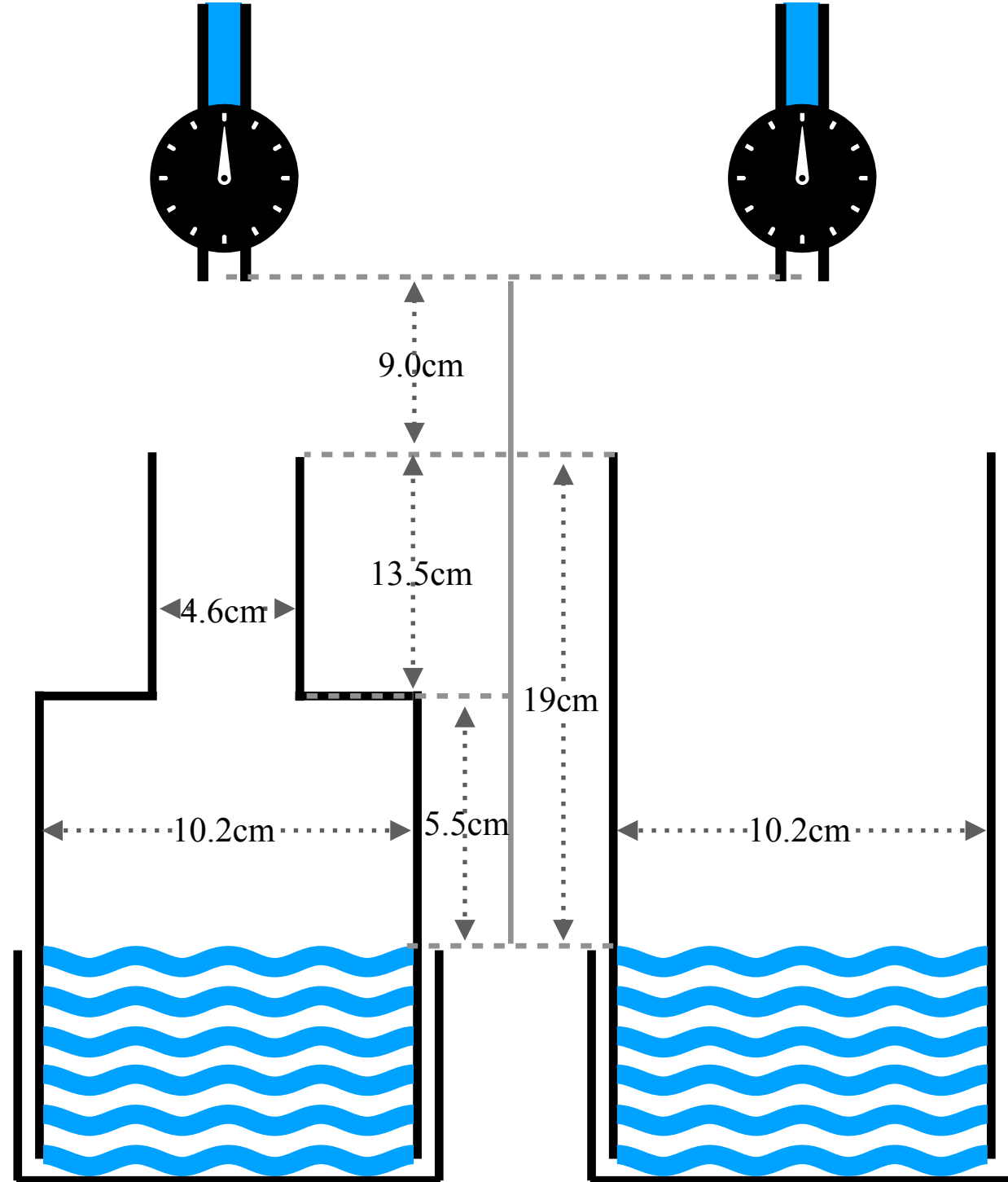
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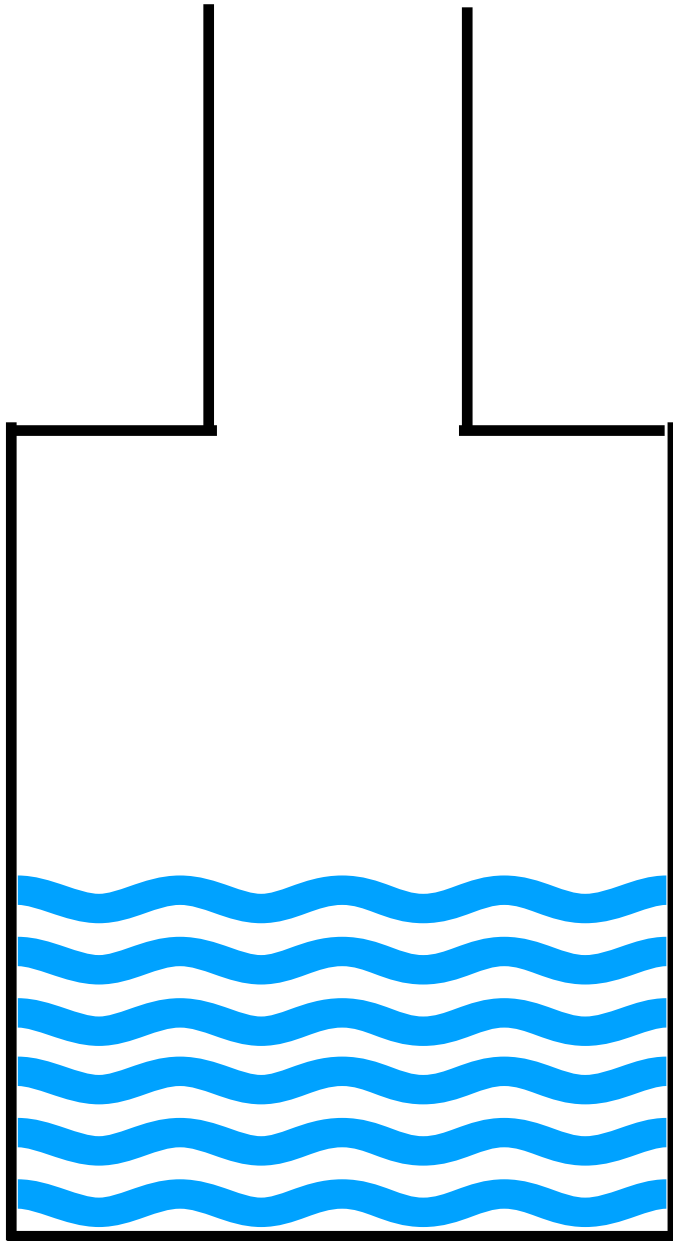




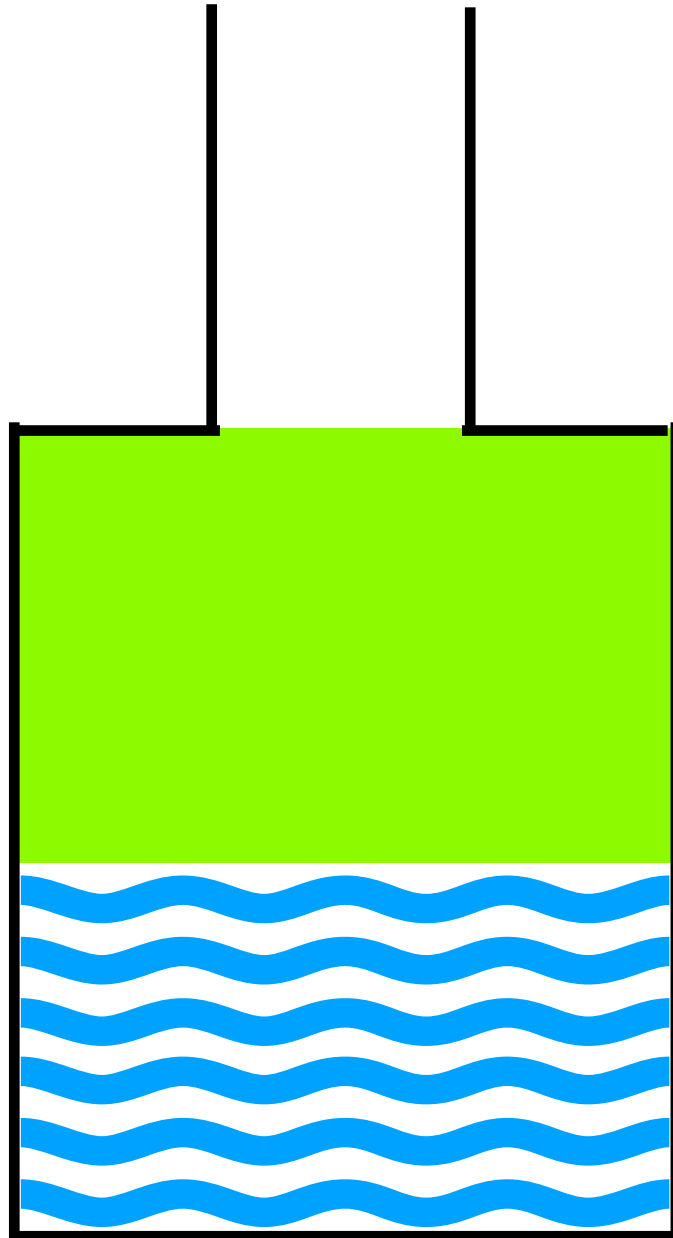




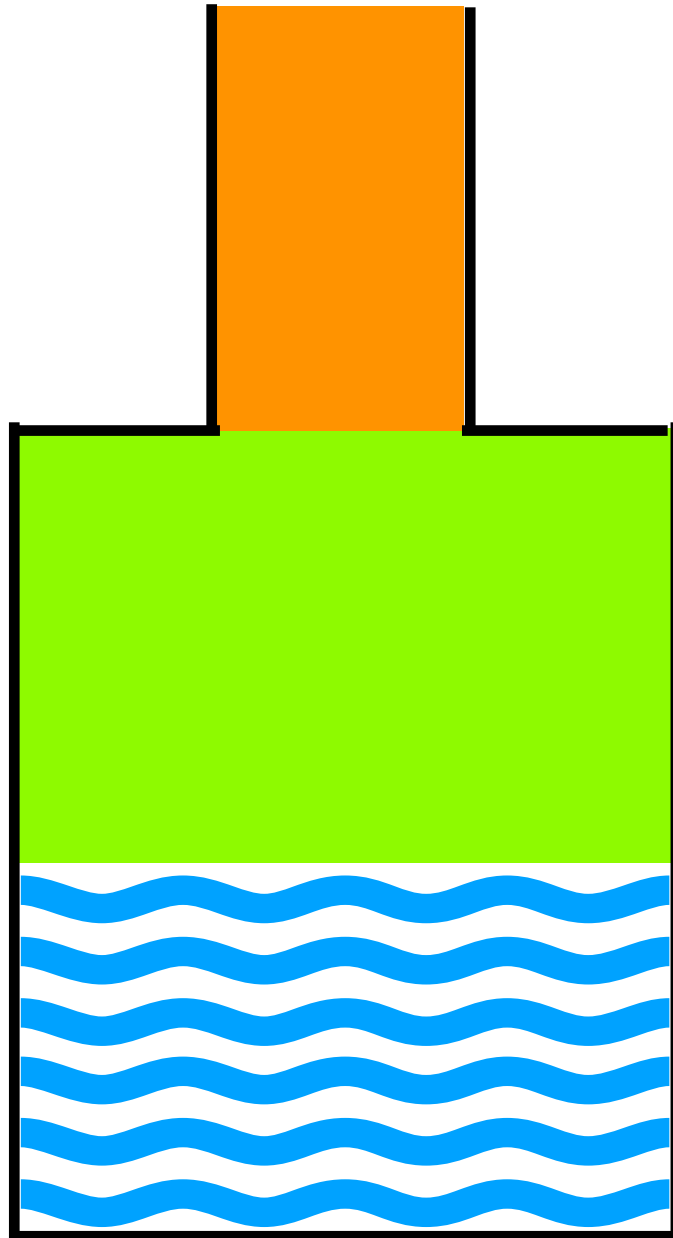
Helmholtz resonance



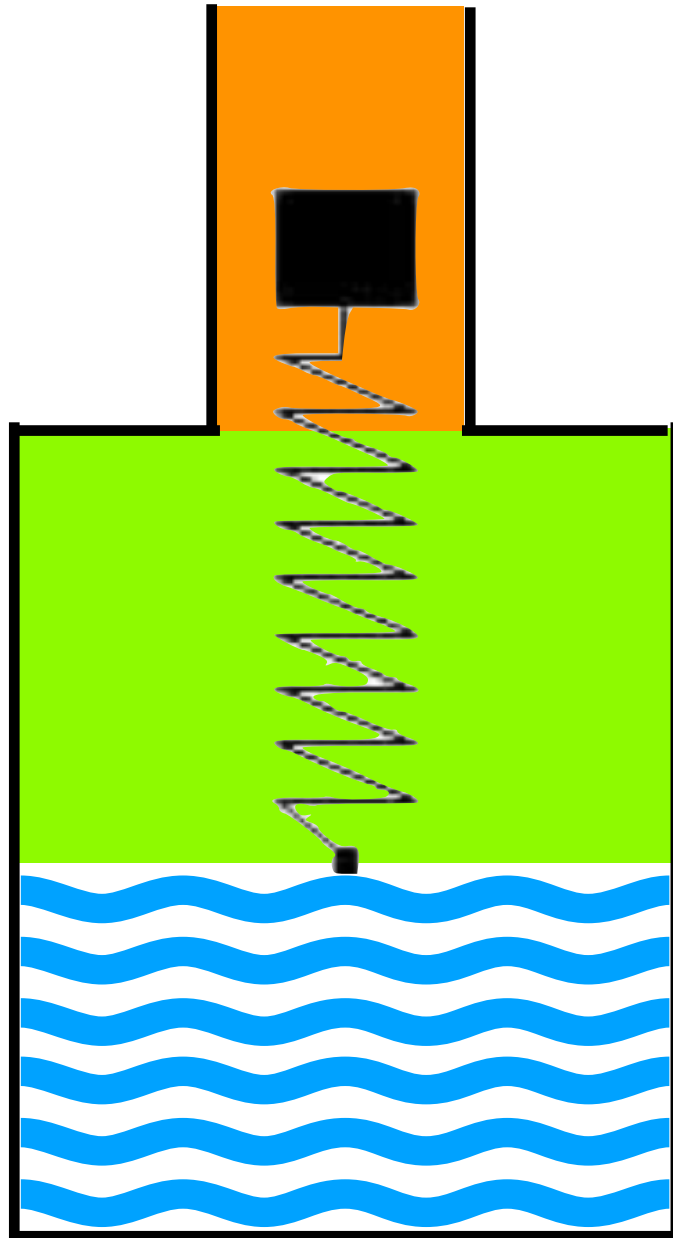
Helmholtz resonance



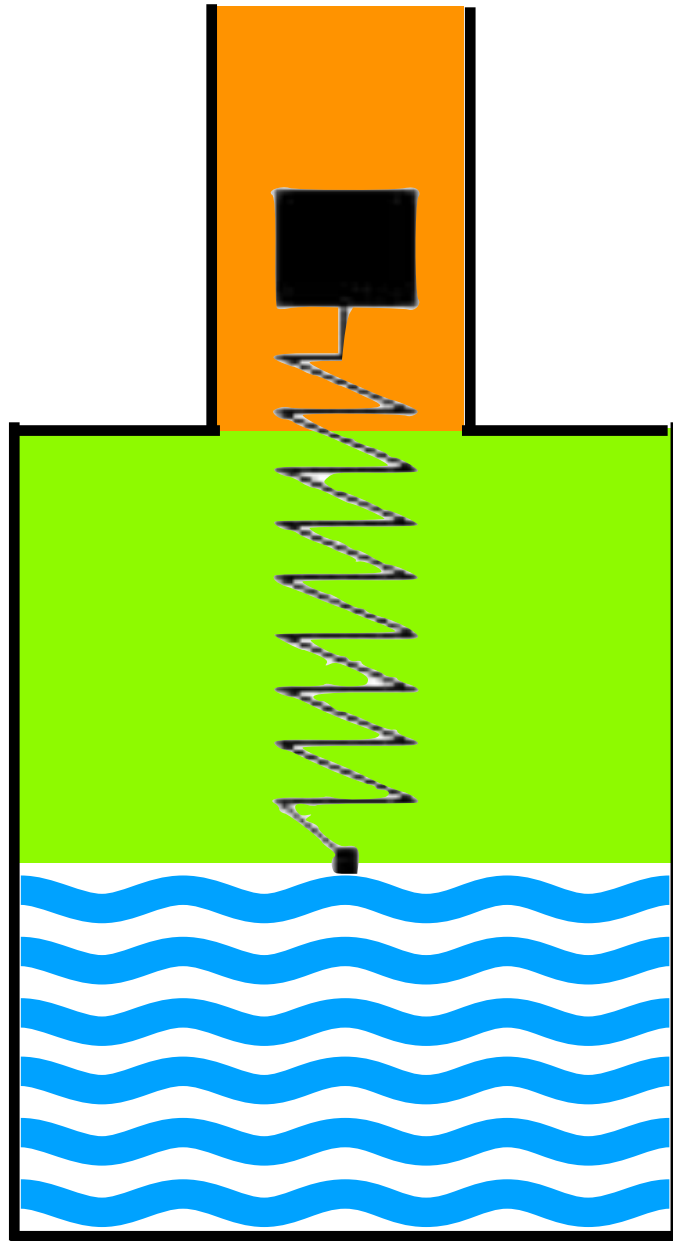
Helmholtz resonance



Helmholtz resonance



Helmholtz resonance



$$f_0 = \frac{c}{2\pi} \cdot \sqrt{\frac{A_{opening}}{V_{air} \cdot l_{neck}}}$$

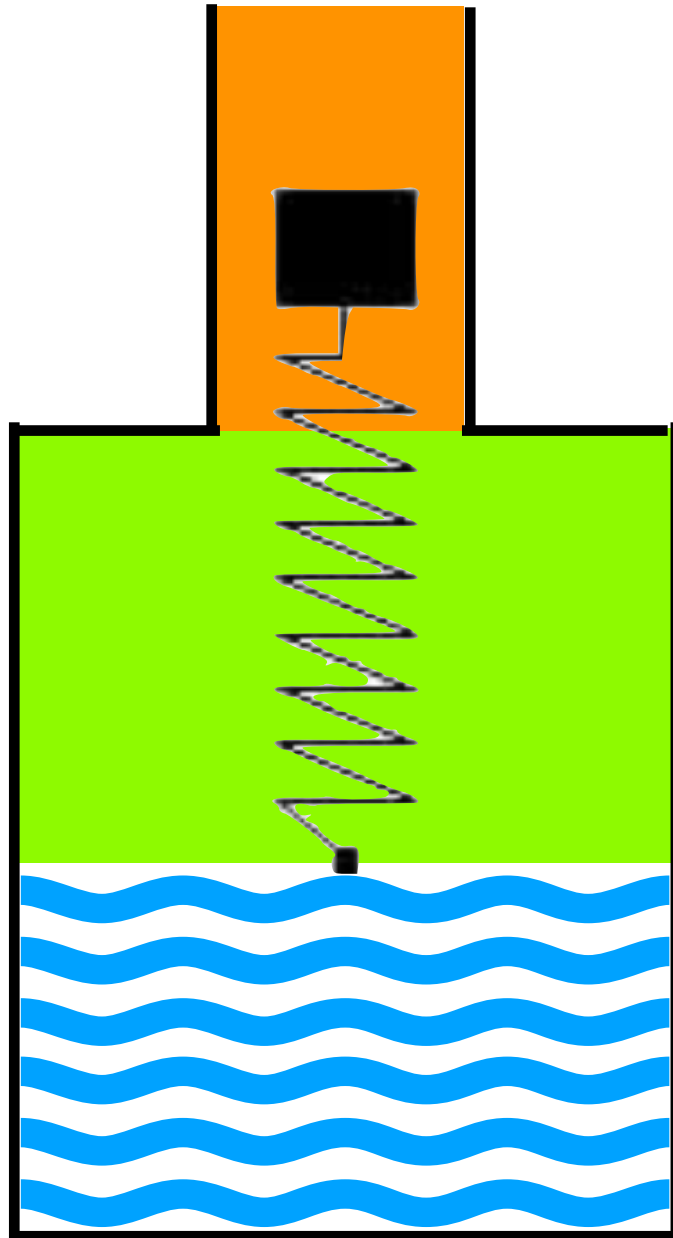
$A_{opening}$ - Area of the opening

V_{air} - Volume of the air in the container

l_{neck} - Length of the neck



Helmholtz resonance



$$f_0 = \frac{343 \frac{m}{s}}{2\pi} \cdot \sqrt{\frac{0.00166 m^2}{0.0449 m^3 \cdot 0.135 m}}$$

$$\approx 26.8 Hz$$

$$f_1 = 3 \cdot f_0 = 80.4 Hz$$

$$f_2 = 5 \cdot f_0 = 134.0 Hz$$

$$f_3 = 7 \cdot f_0 = 187.6 Hz$$

$$f_4 = 9 \cdot f_0 = 241.2 Hz$$

...

$A_{opening}$ - Area of the opening

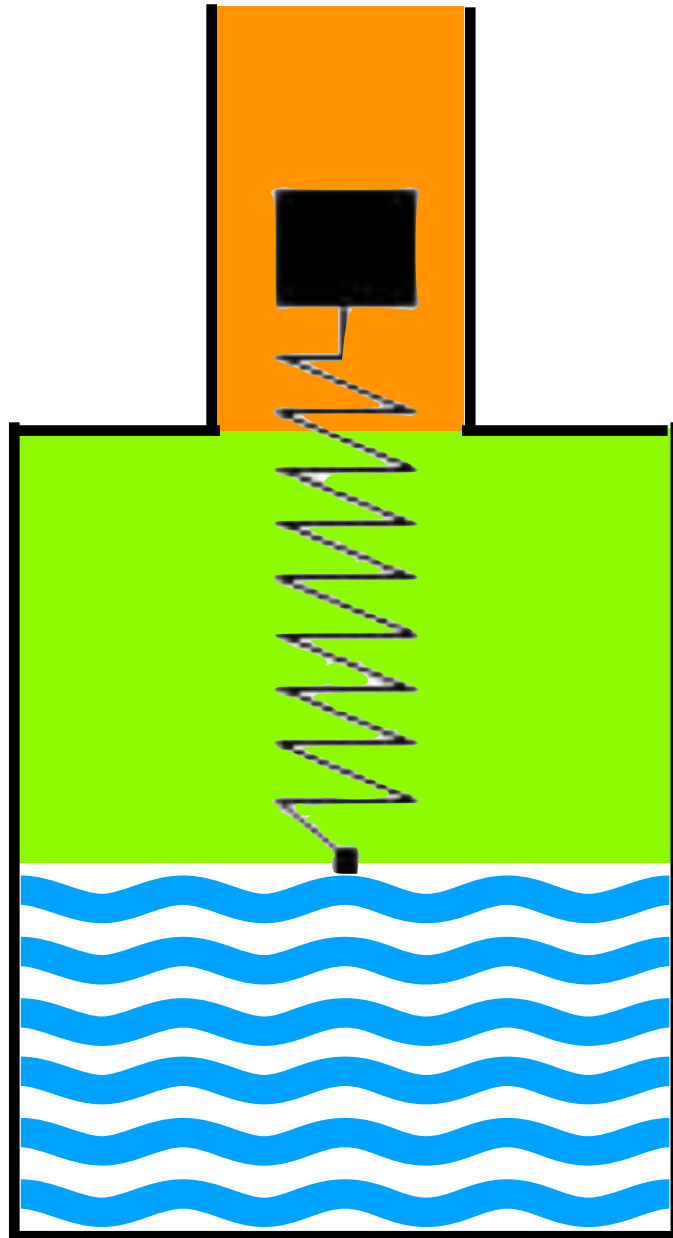
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Helmholtz resonance



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...

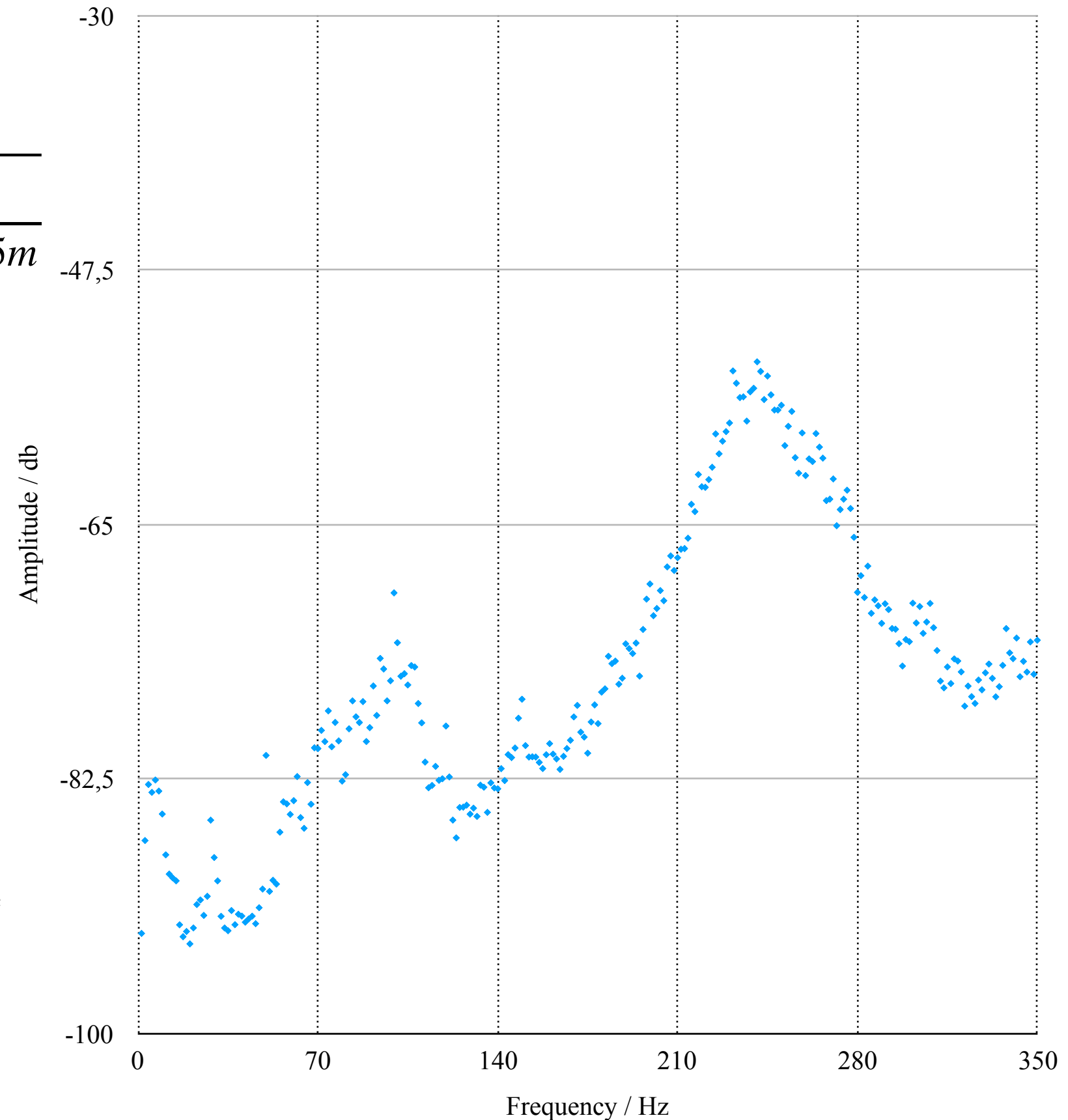
$$f_0 = \frac{c}{2\pi} \cdot \sqrt{\frac{A_{opening}}{V_{air} \cdot l_{neck}}}$$

$A_{opening}$ - Area of the opening

V_{air} - Volume of the air in the container

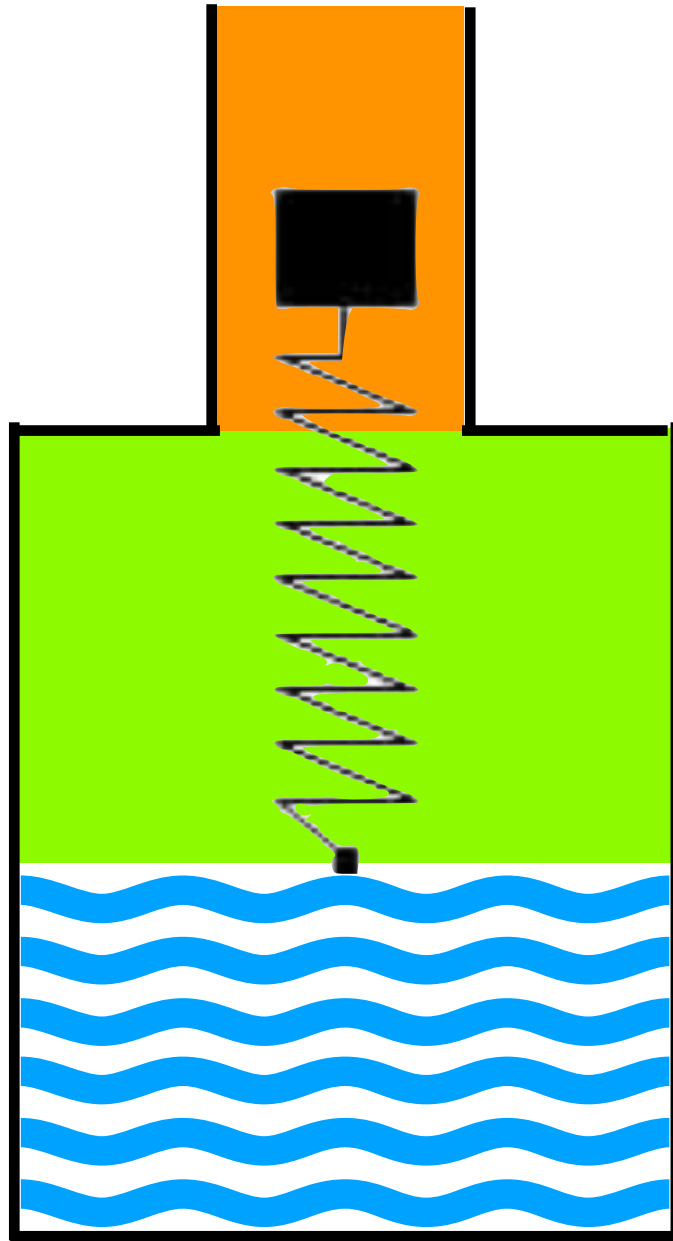
l_{neck} - Length of the neck

× with extra tube





Helmholtz resonance



$$f_0 = \frac{343 \frac{m}{s}}{2\pi} \cdot \sqrt{\frac{0.00166 m^2}{0.0449 m^3 \cdot 0.135 m}}$$

$$\approx 26.8 Hz$$

$$f_1 = 3 \cdot f_0 = 80.4 Hz$$

$$f_2 = 5 \cdot f_0 = 134.0 Hz$$

$$f_3 = 7 \cdot f_0 = 187.6 Hz$$

$$f_4 = 9 \cdot f_0 = 241.2 Hz$$

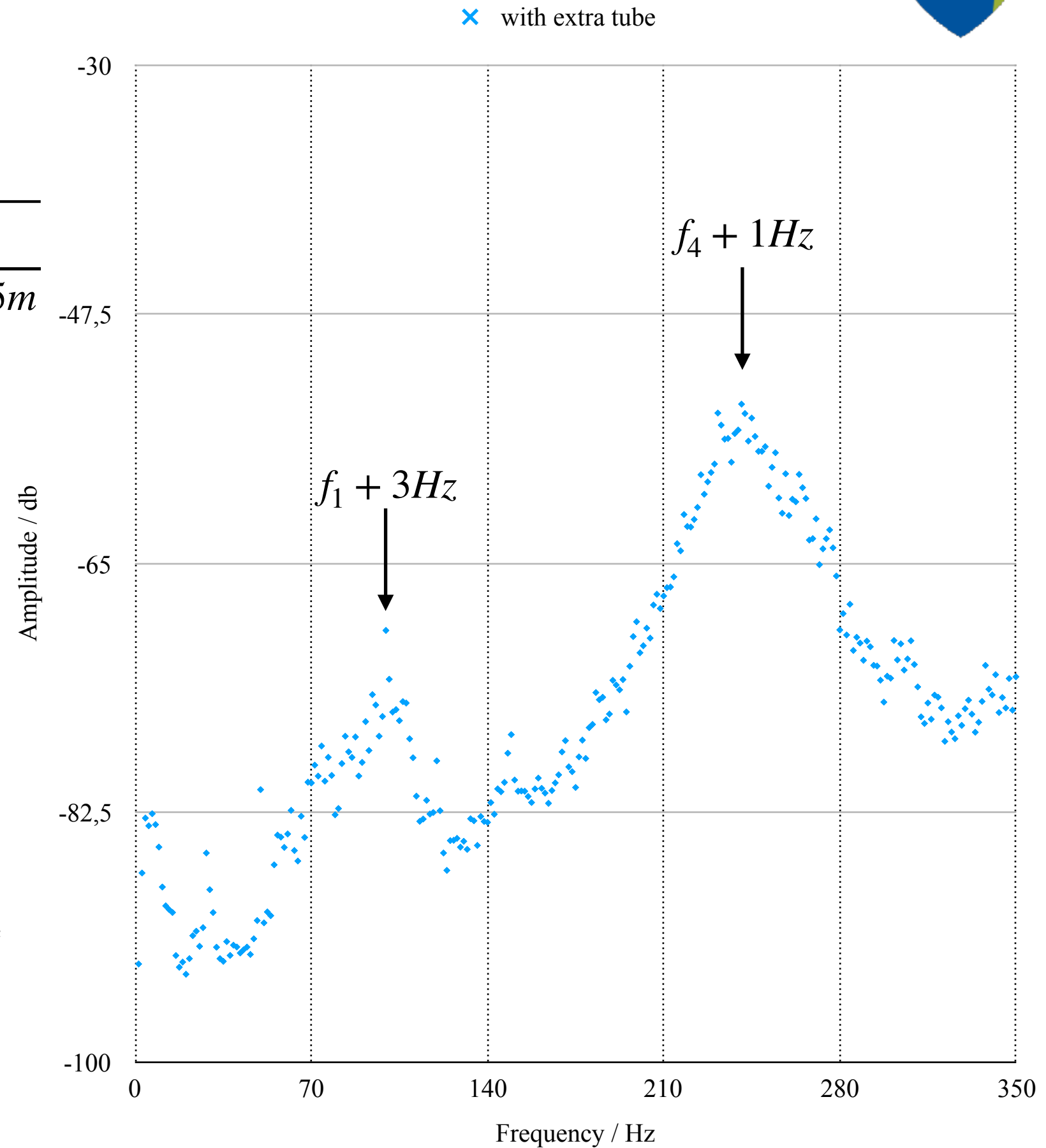
$$\dots$$

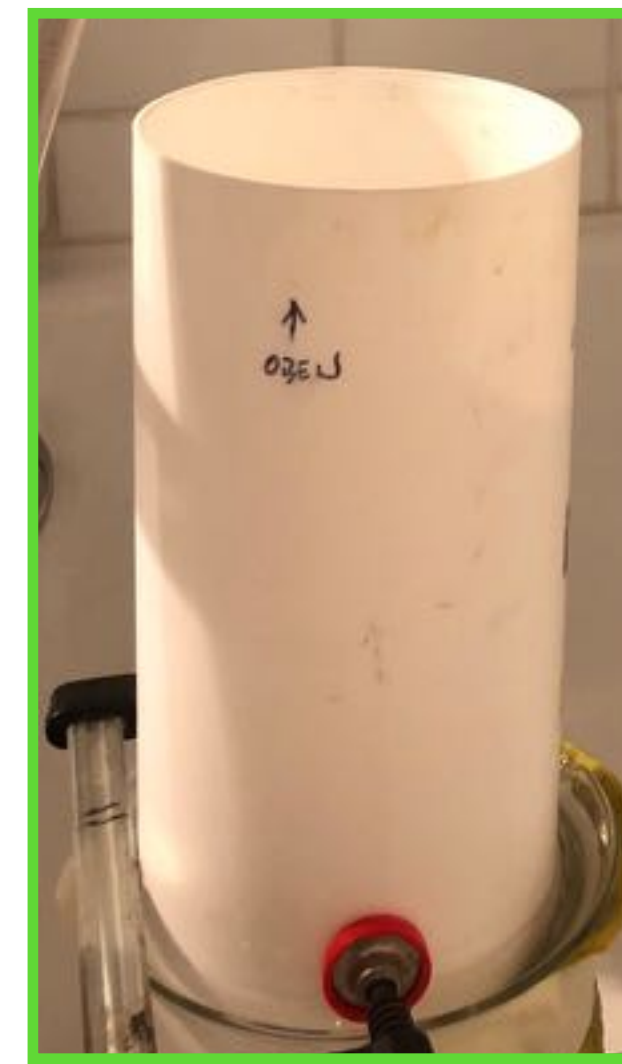
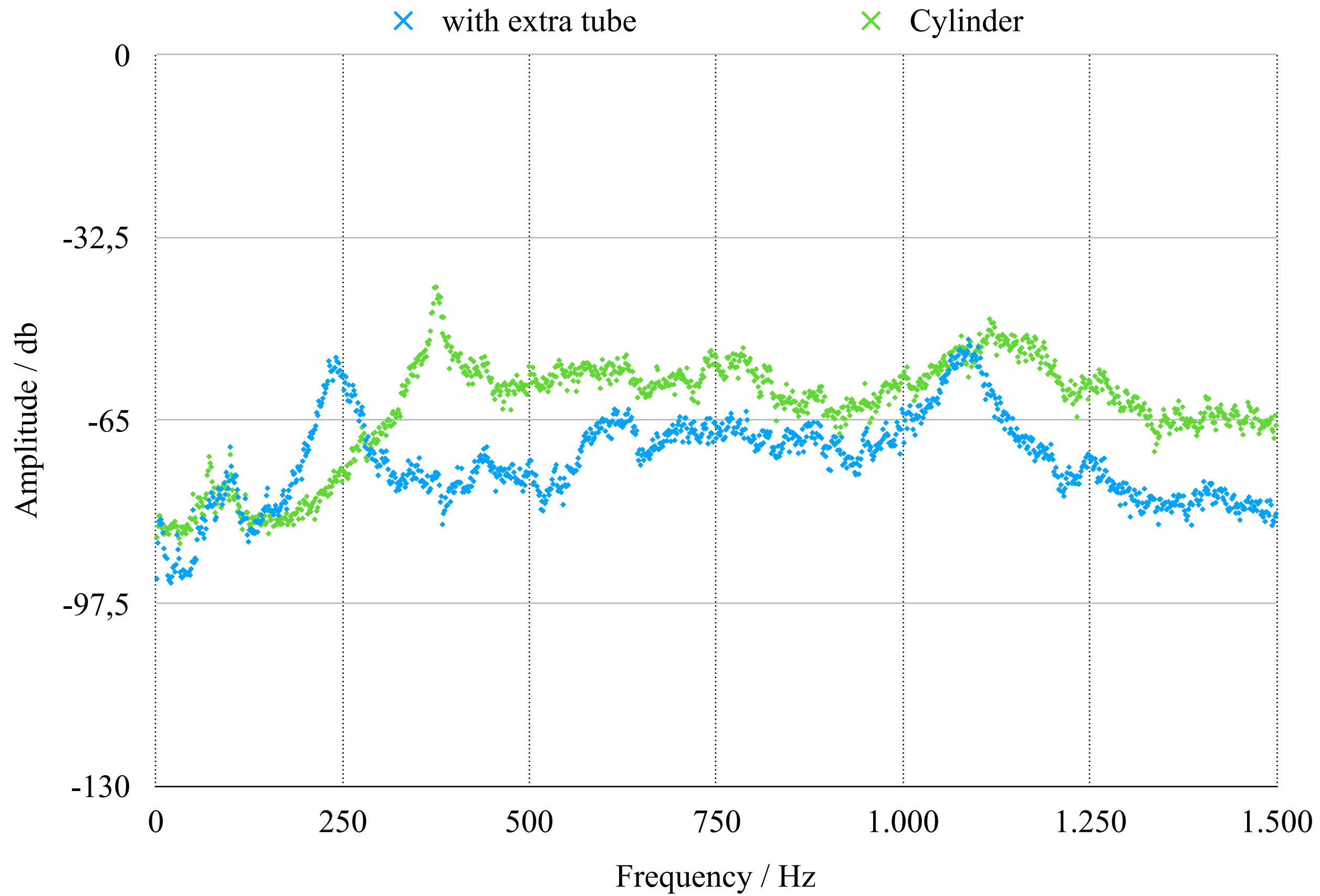
$$f_0 = \frac{c}{2\pi} \cdot \sqrt{\frac{A_{opening}}{V_{air} \cdot l_{neck}}}$$

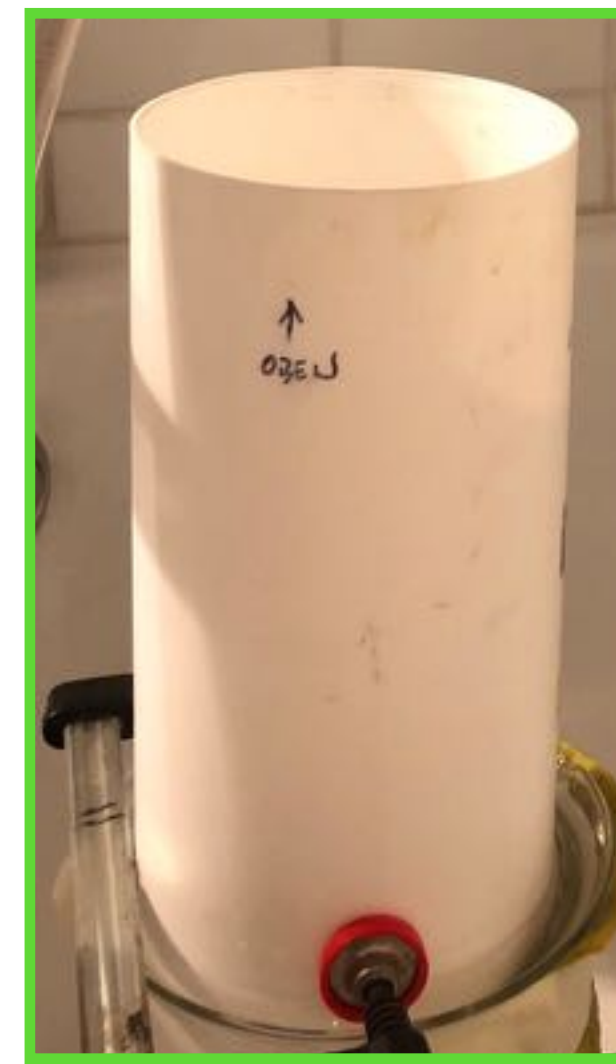
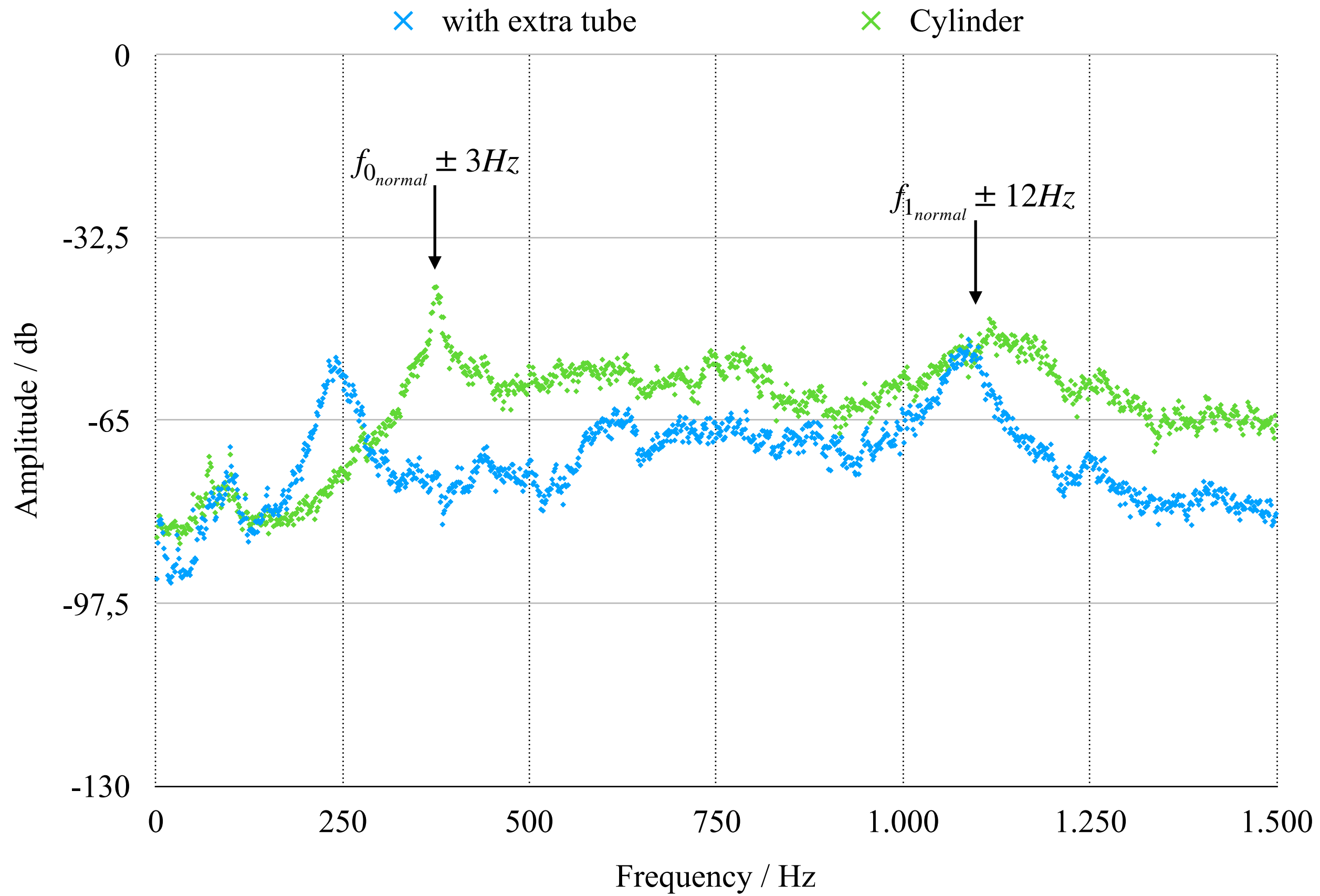
$A_{opening}$ - Area of the opening

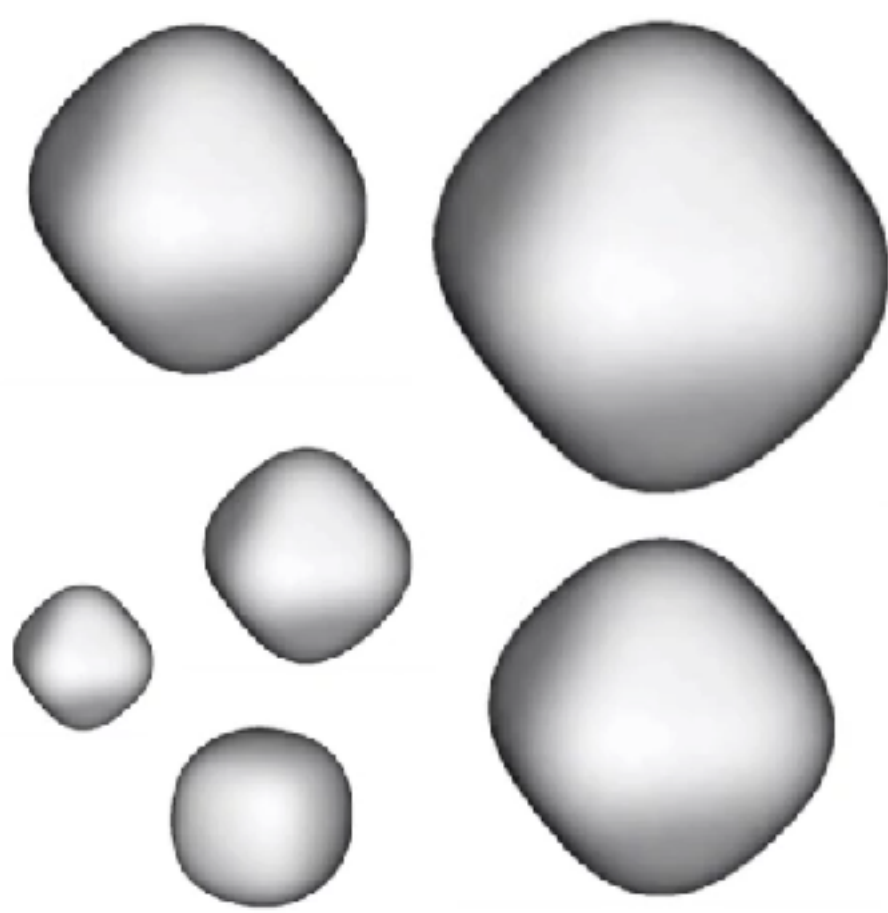
V_{air} - Volume of the air in the container

l_{neck} - Length of the neck

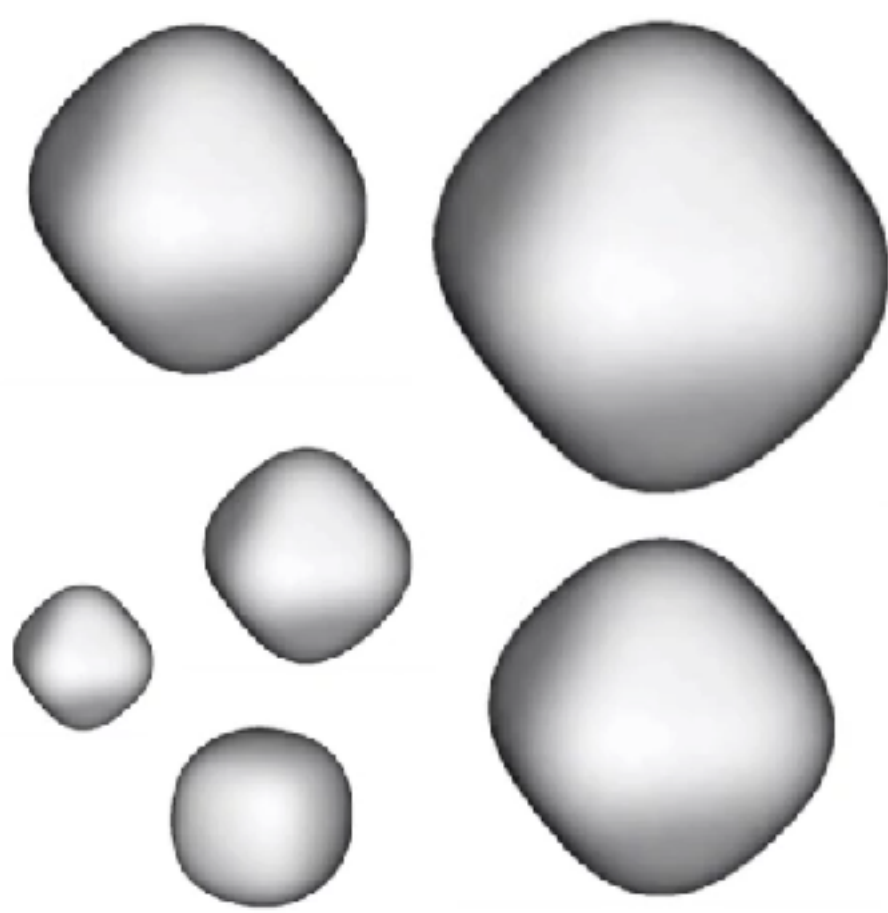






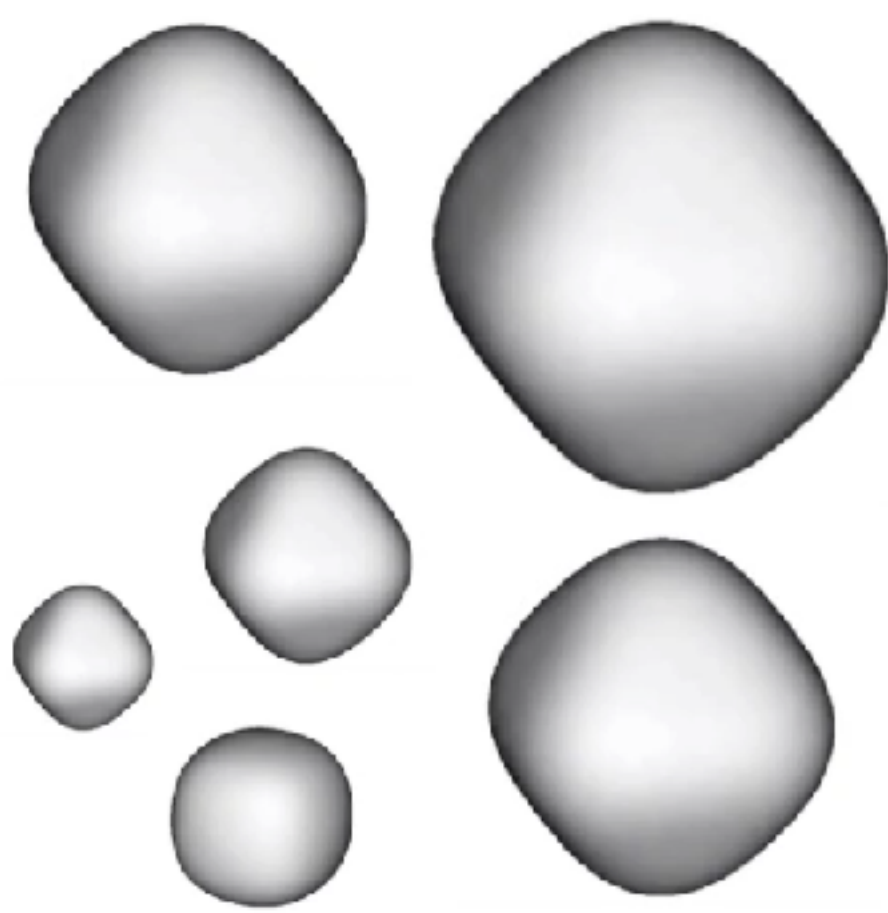


Conclusion



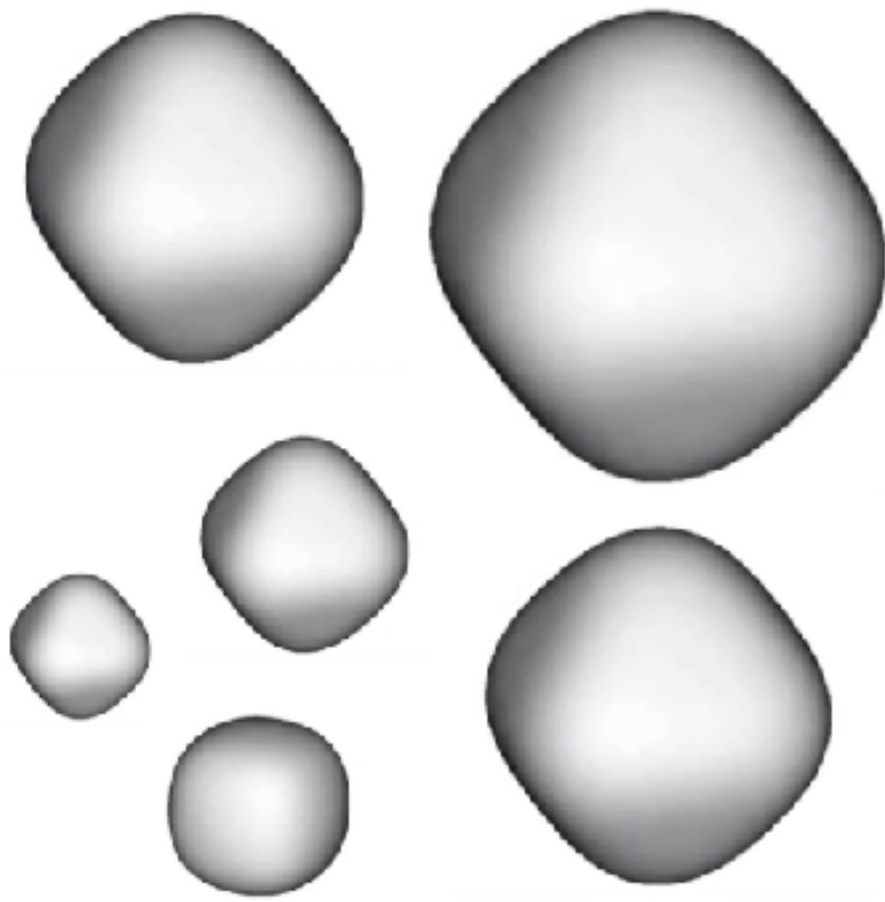
Conclusion

Conclusion



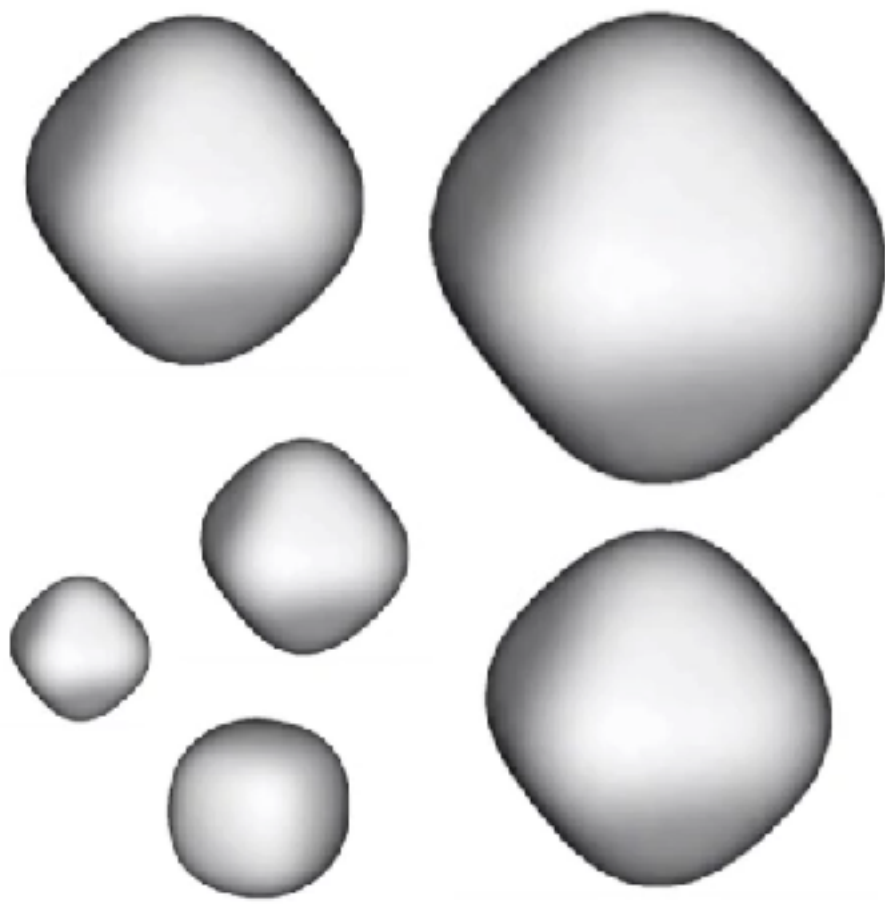
**Bubbles with
different radii
oscillate with
different frequencies**

Conclusion

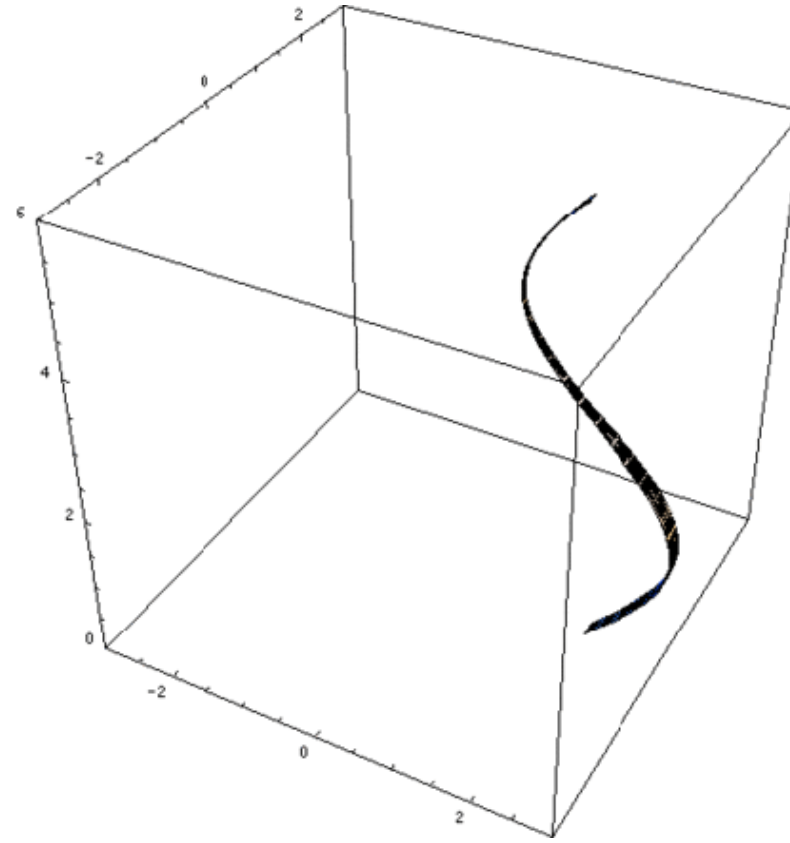


**Bubbles with
different radii
oscillate with
different frequencies**

**Larger bubbles
oscillate with lower
frequencies, which
are louder**

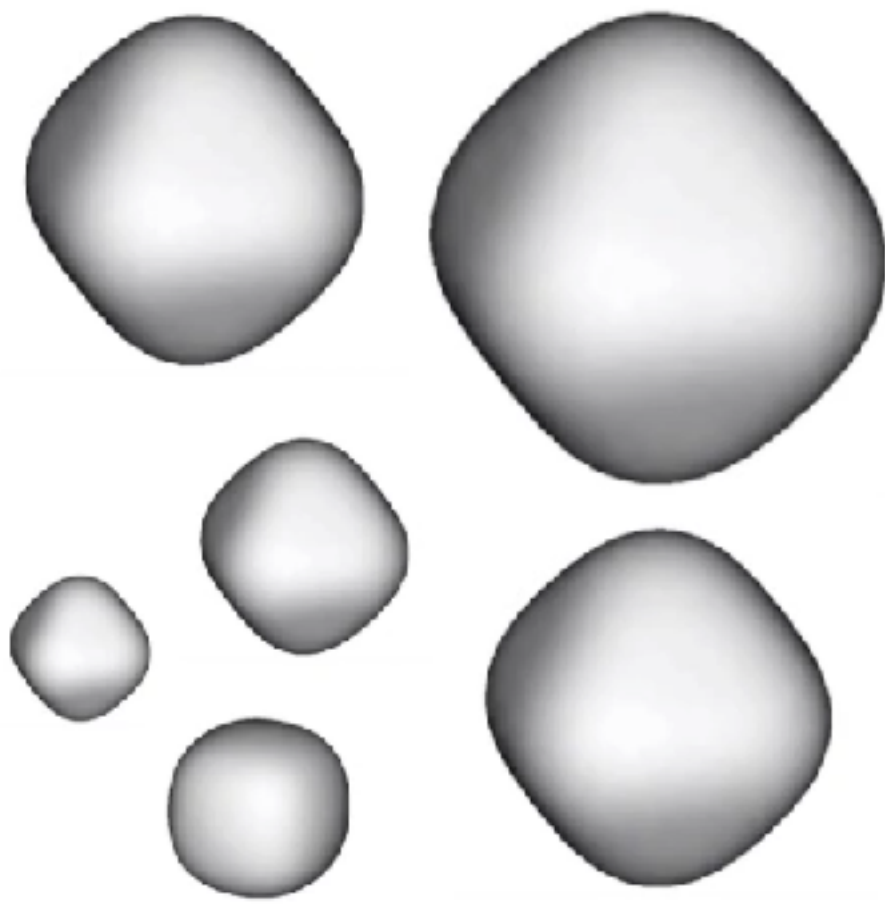


Conclusion

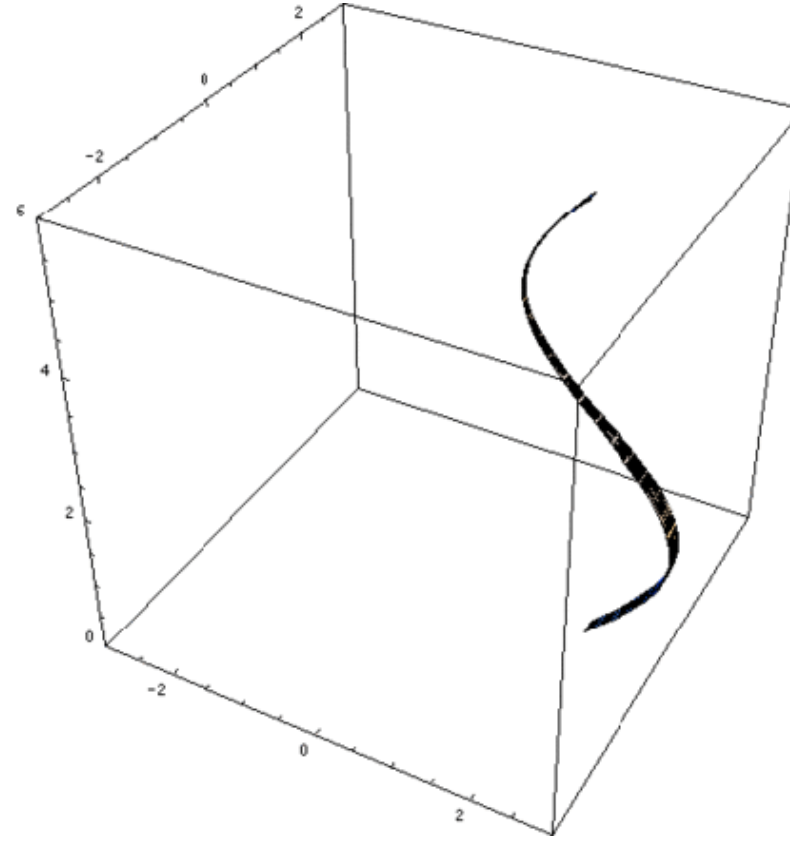


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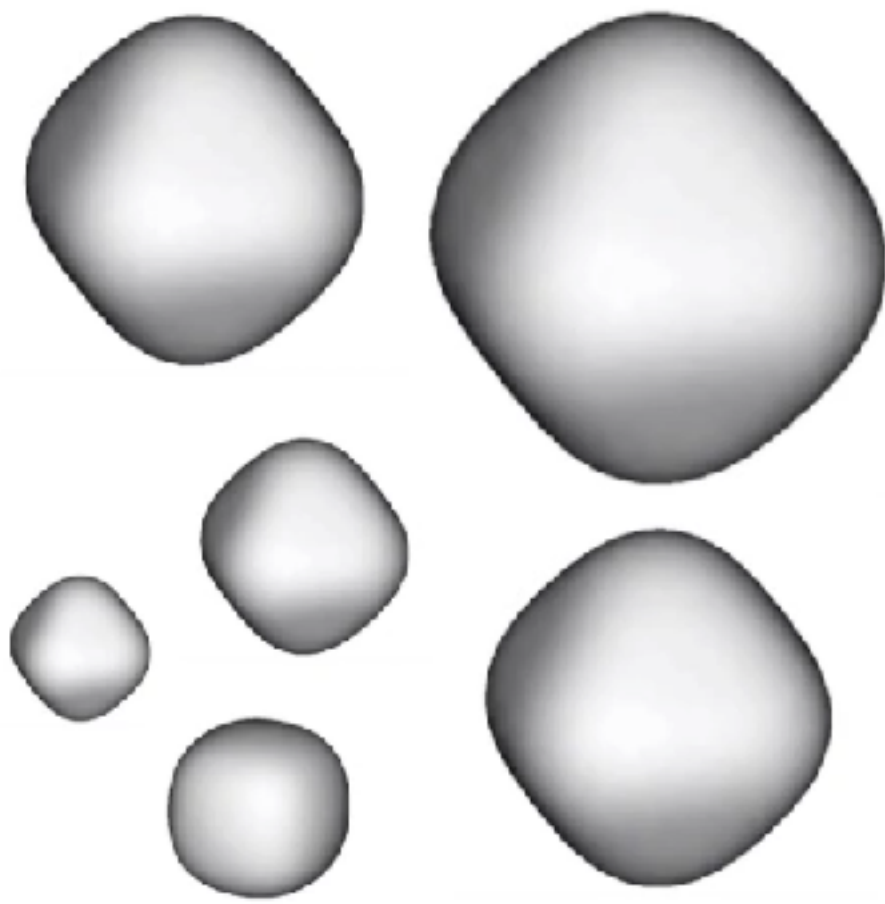
Conclusion



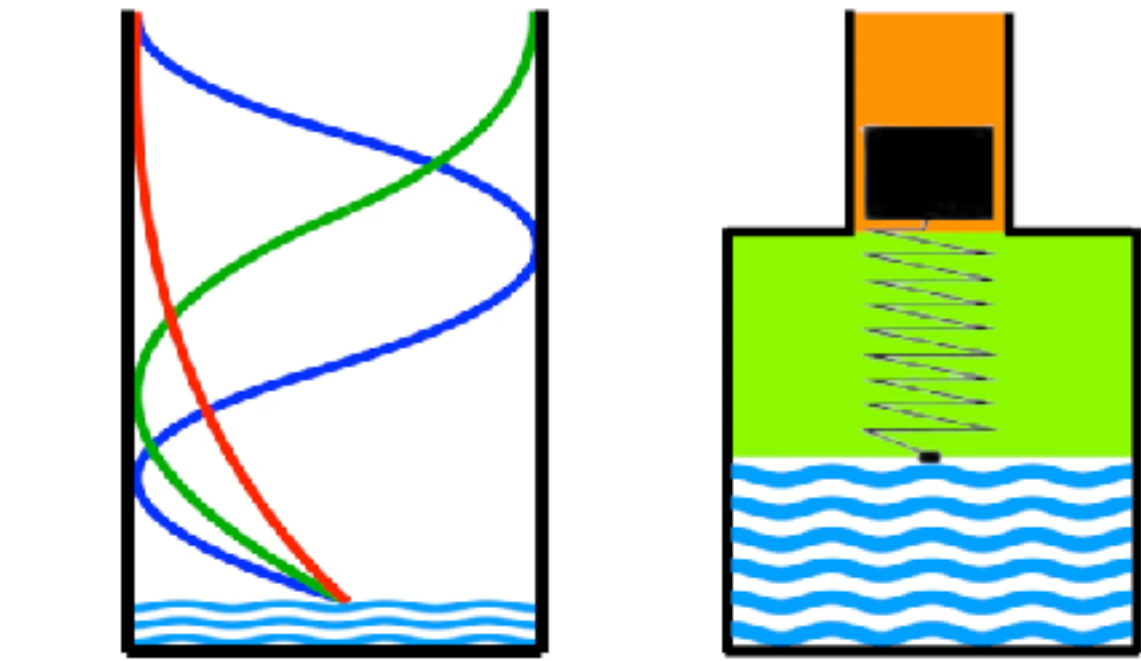
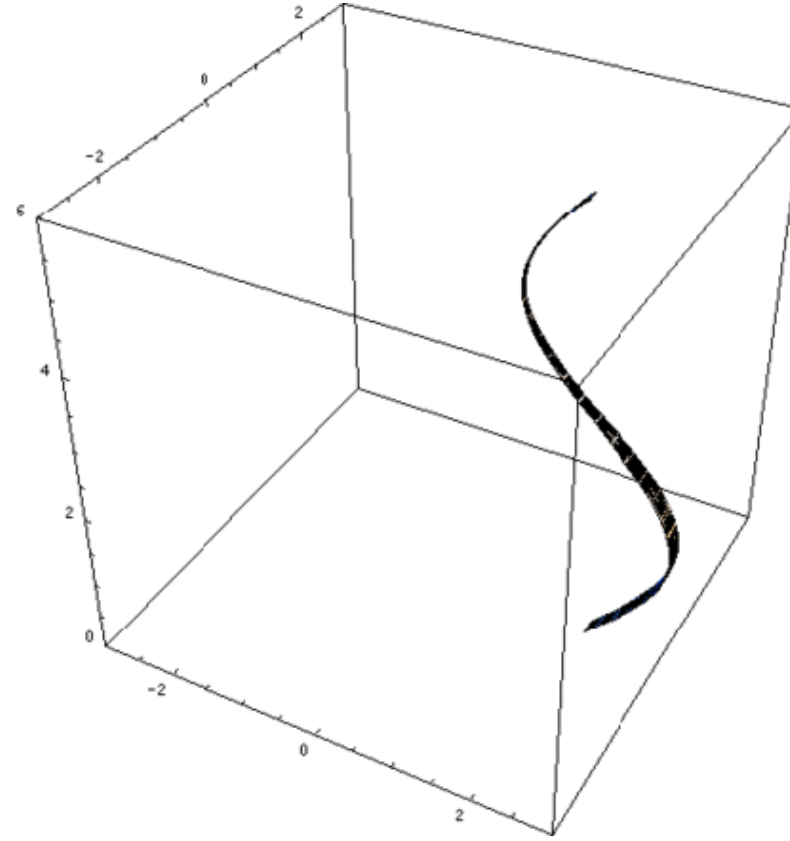
**Bubbles with
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**For every bottle a
prediction for the
cavity at any
given moment can
be calculated by
the shape of the
bottle**



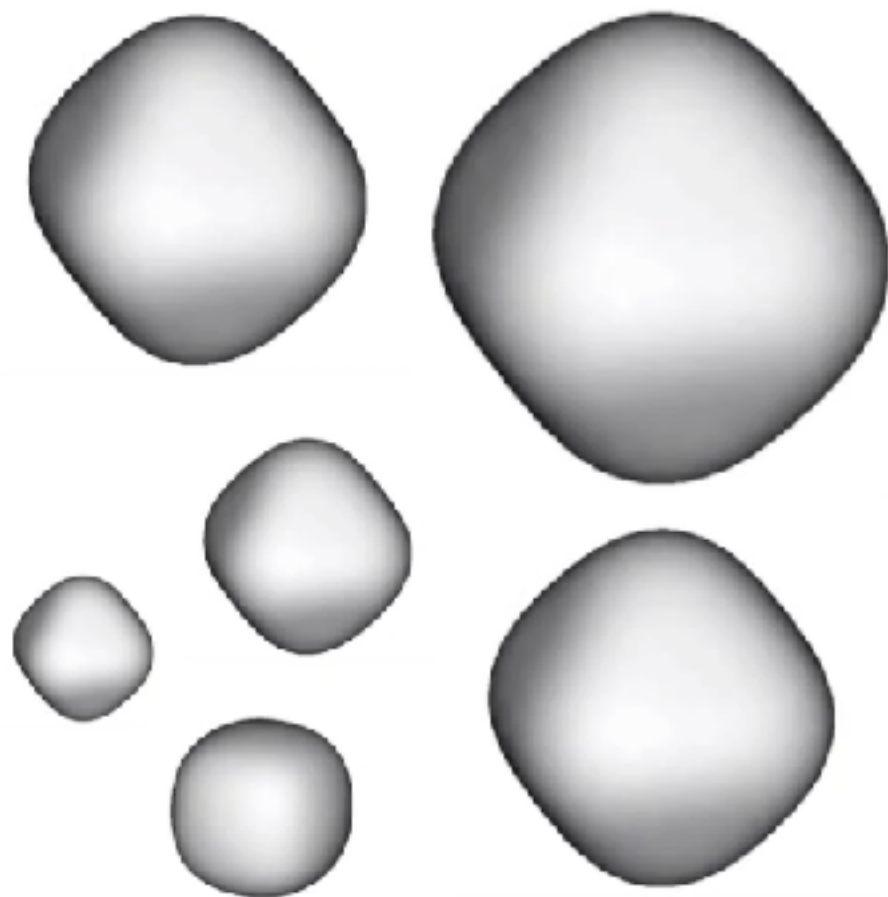
Conclusion



Bubbles with different radii oscillate with different frequencies

Larger bubbles oscillate with lower frequencies, which are louder

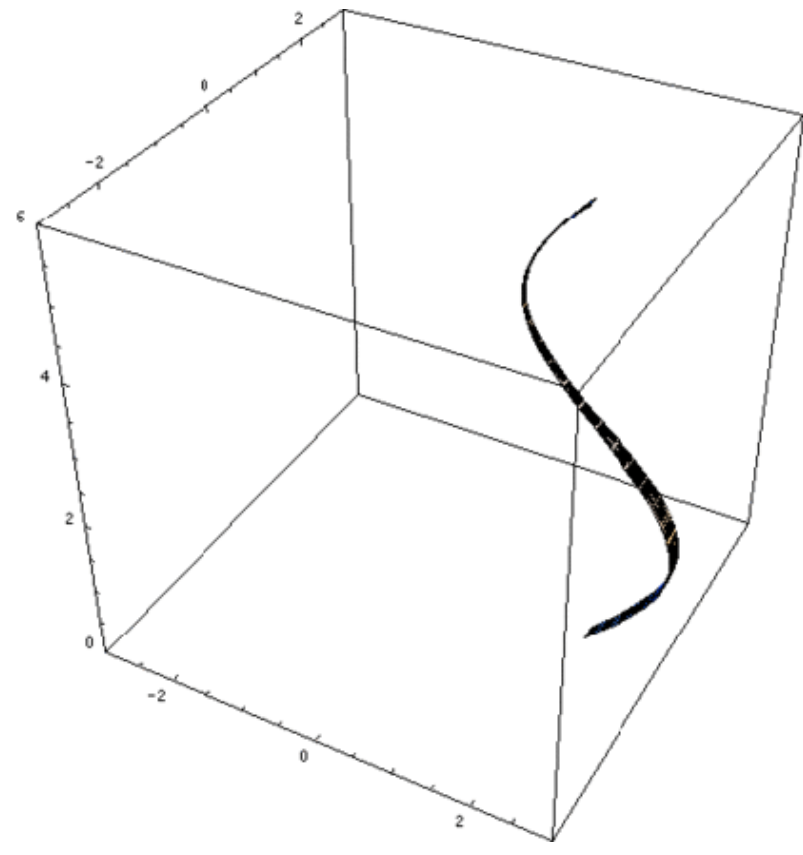
For every bottle a prediction for the cavity at any given moment can be calculated by the shape of the bottle



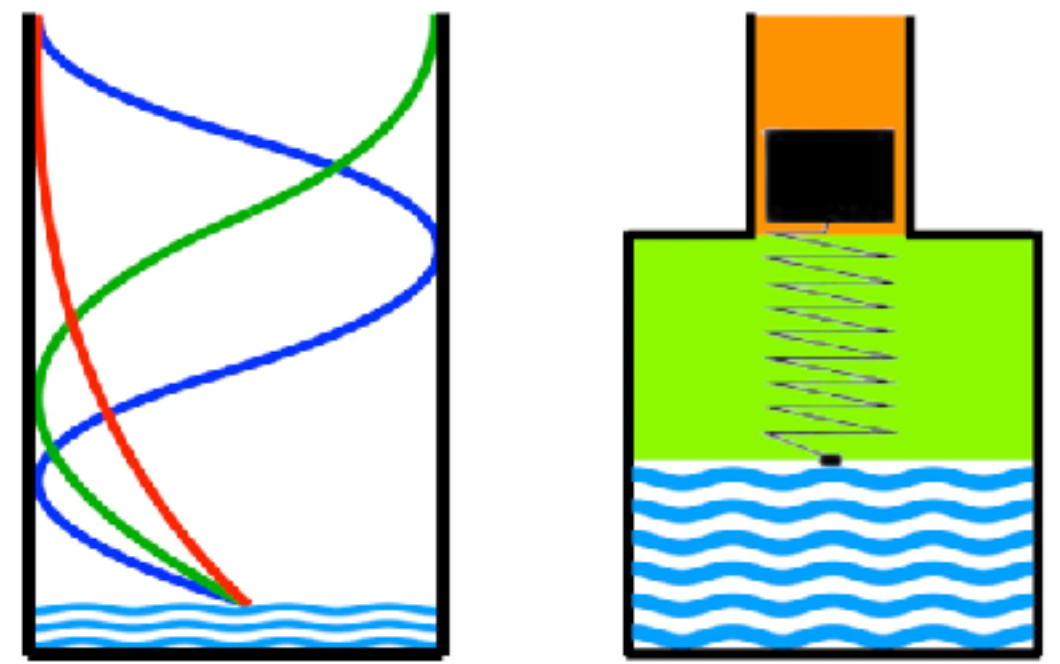
Bubbles with different radii oscillate with different frequencies

Larger bubbles oscillate with lower frequencies, which are louder

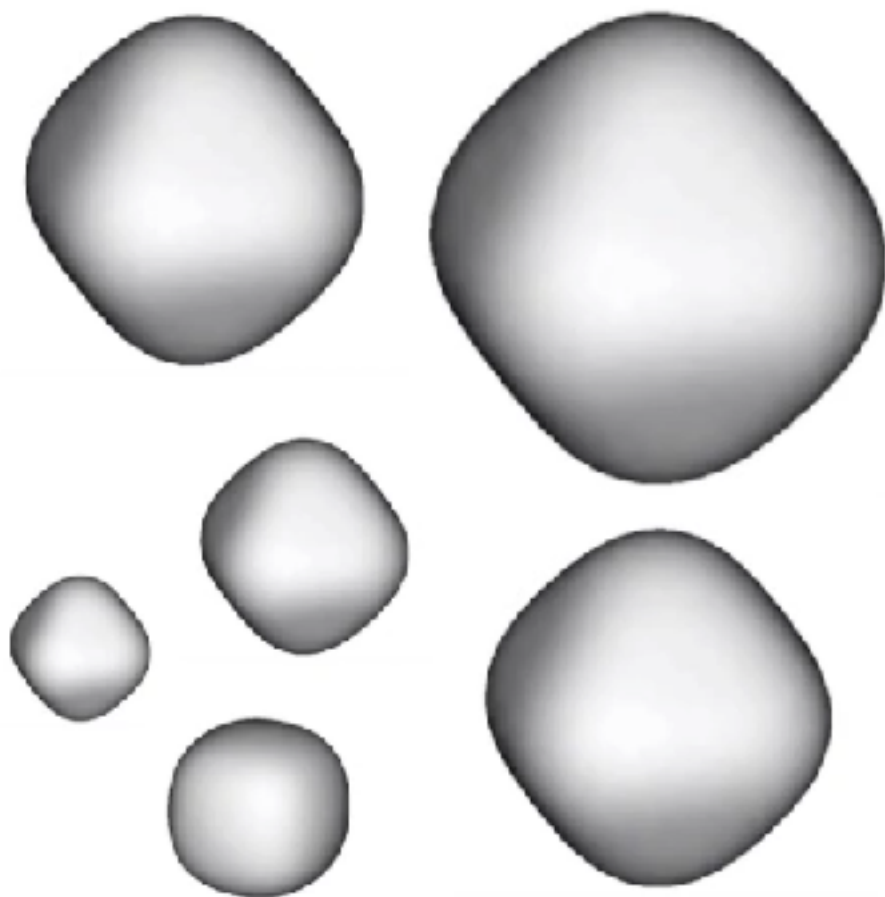
Conclusion



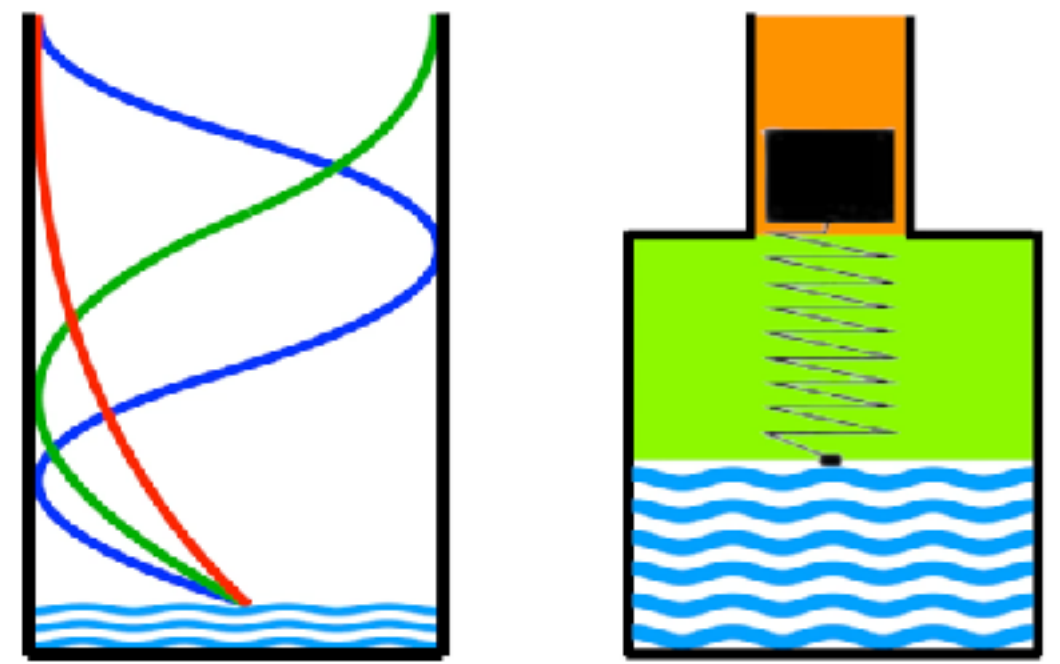
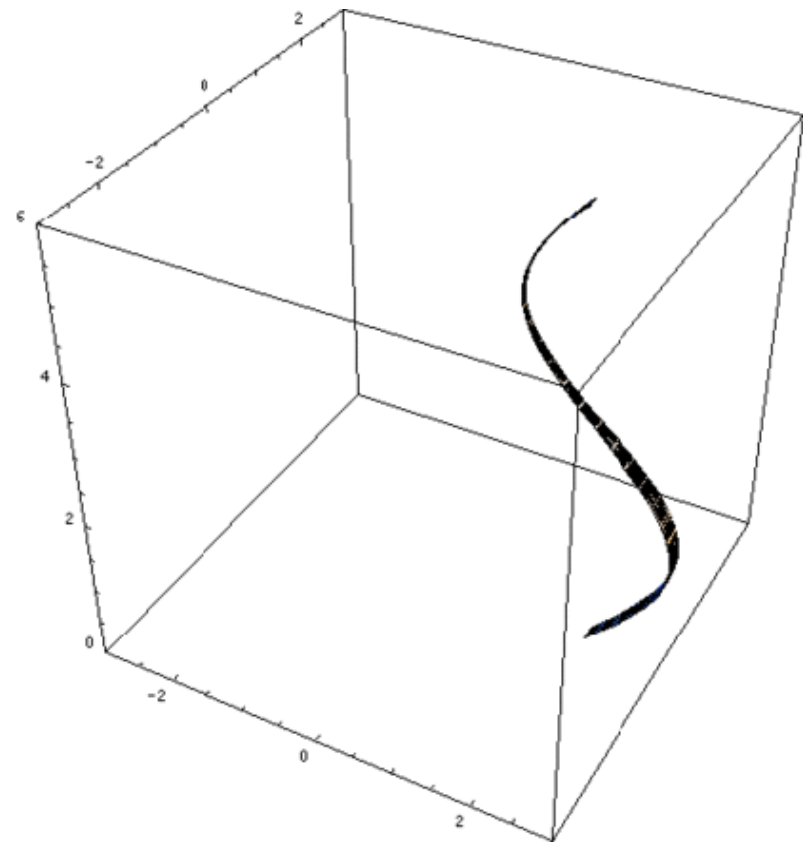
For every bottle a prediction for the cavity at any given moment can be calculated by the shape of the bottle



Standing waves inside the cavity amplify certain frequencies



Conclusion

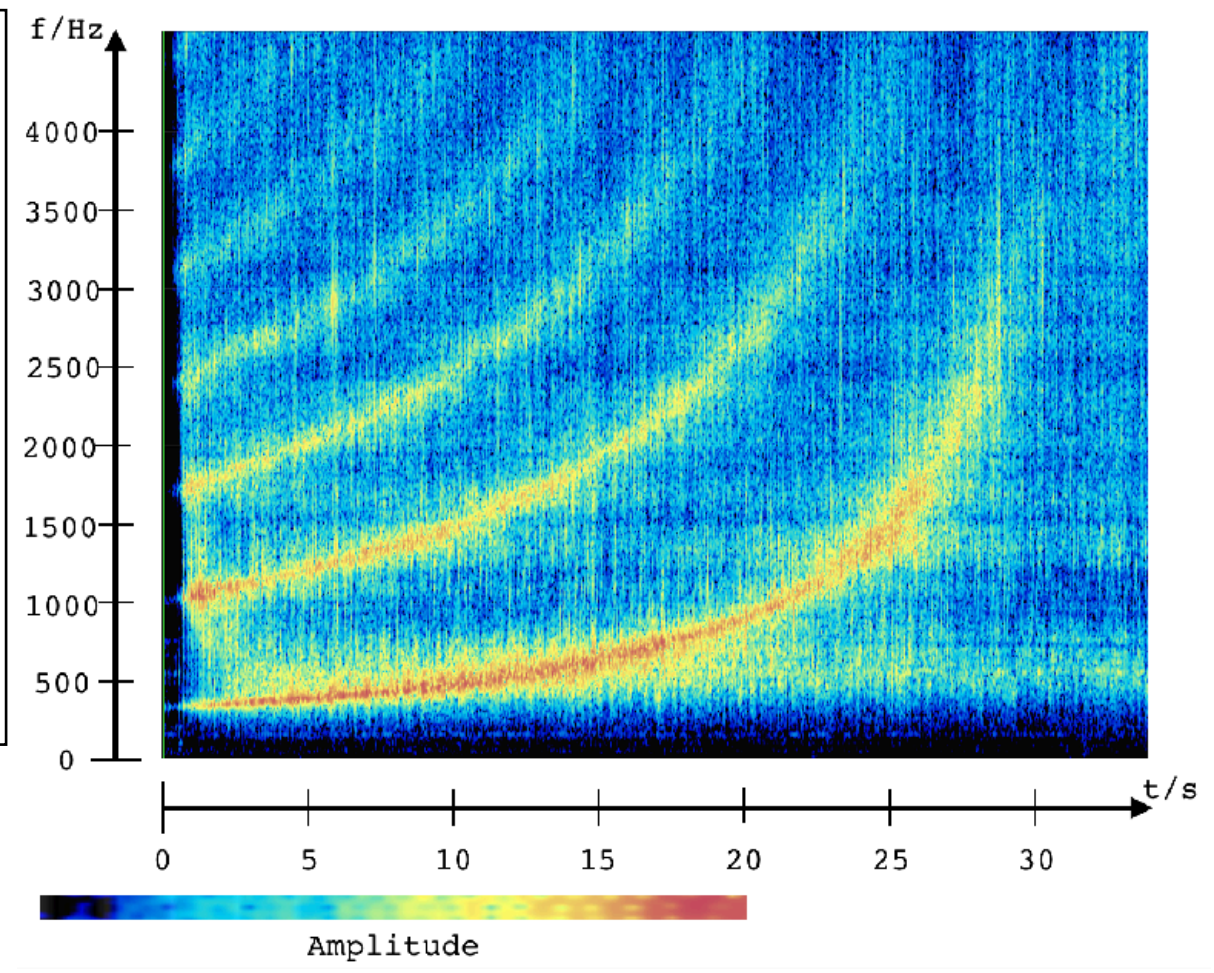


Standing waves inside the cavity amplify certain frequencies

Bubbles with different radii oscillate with different frequencies

Larger bubbles oscillate with lower frequencies, which are louder

For every bottle a prediction for the cavity at any given moment can be calculated by the shape of the bottle



Thank you for your attention.

References

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Jens Eggers and Emmanuel Villermaux. Physics of liquid jets (2007)
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Yves Brechet and Zoltan Neda. On the circular hydraulic jump (1999)

$$h_w(t)$$

Mass conservation says that the rate of mass accumulated in the tank equals the mass flow rate \dot{m}_{in} into the tank:

$$\frac{dM_{tank}}{dt} = \dot{m}_{in}$$

where the mass in the tank at time t is given by

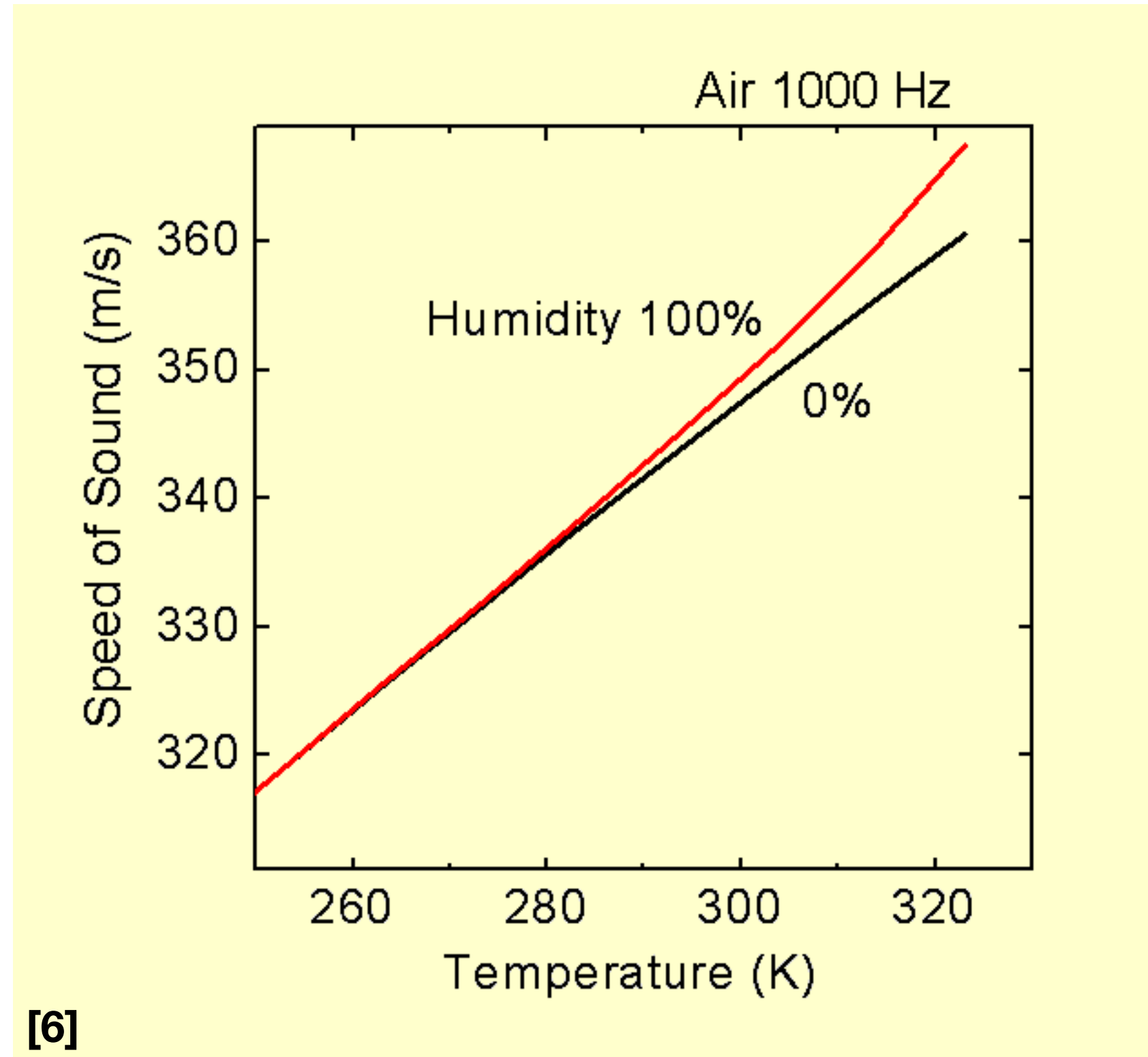
$$M_{tank} = \rho\pi \int_0^{h_w(t)} r^2(z) dz$$

and $r(z)$ is the radius of the solid of tank shape as a function of height z , is the height $h(t)$ of the fluid.

Therefore $h(t)$ and $\frac{dh_w}{dt}$ are calculable by solving

$$\rho\pi \frac{d}{dt} \int_0^{h_w(t)} r^2(z) dz = \dot{m}_{in}$$

Influence of temperature



[6]

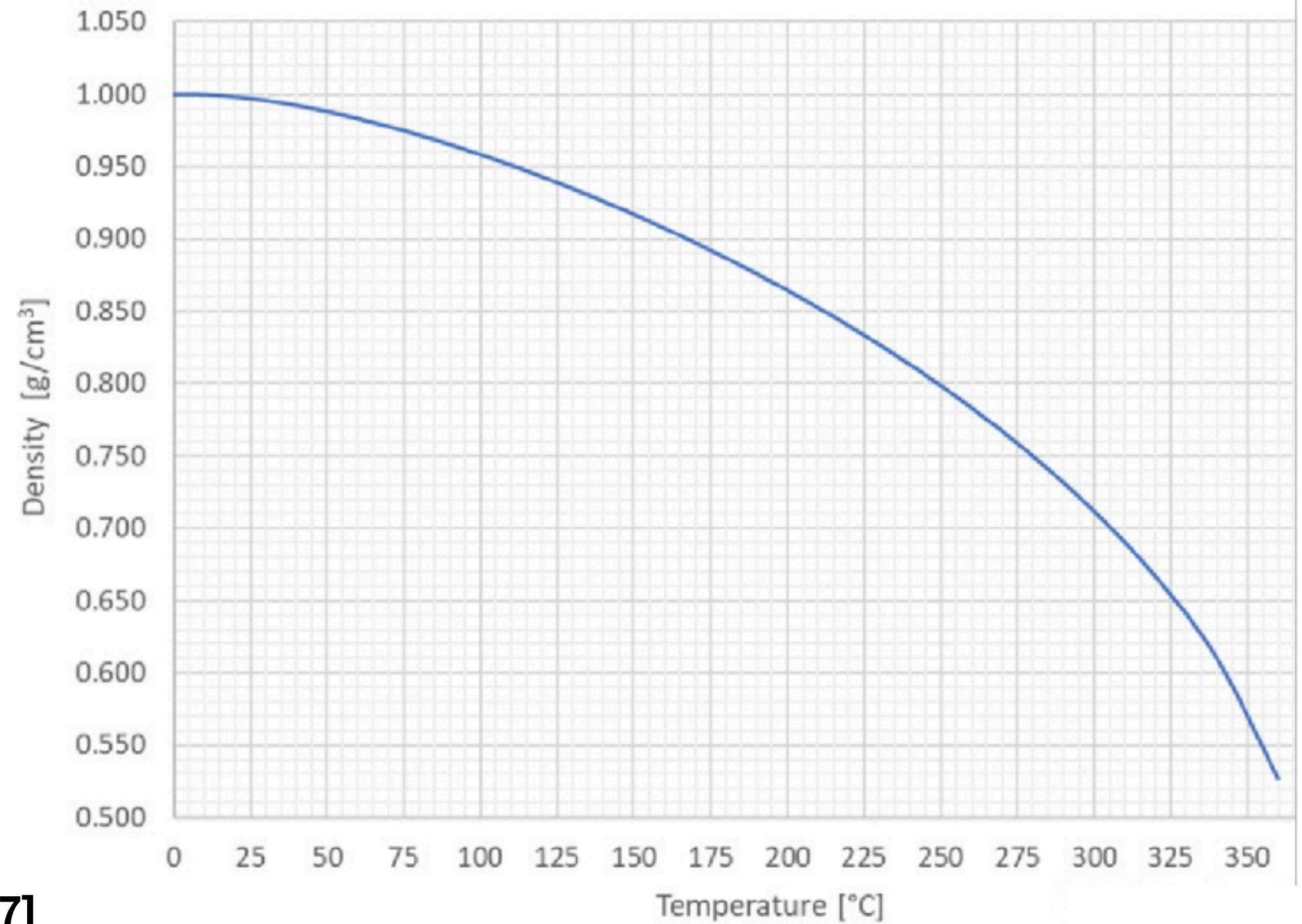
[6] <http://pages.mtu.edu/~suits/SpeedofSound.html>

Influence of temperature

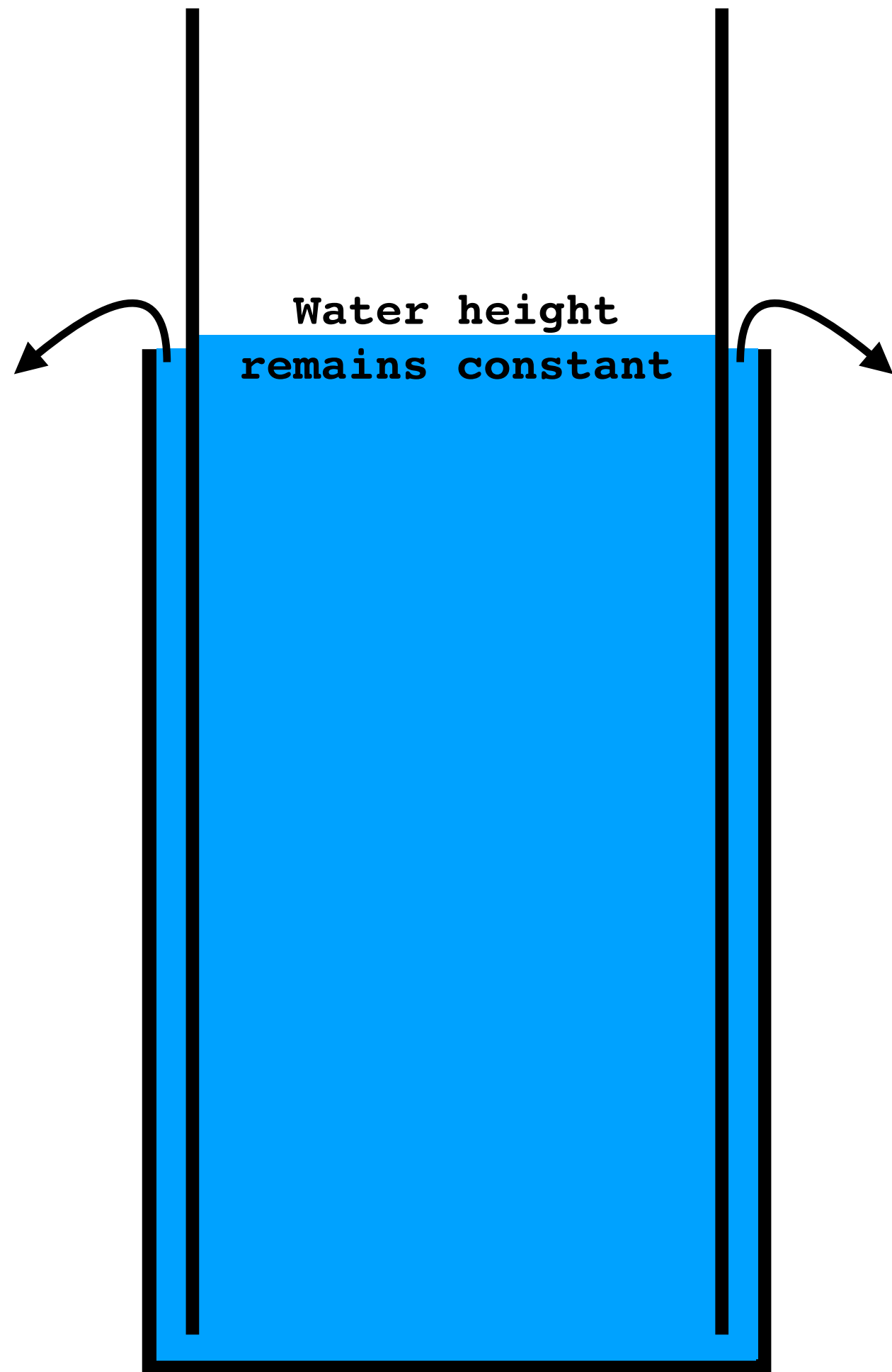


[7]

Density of water
at water saturation pressure

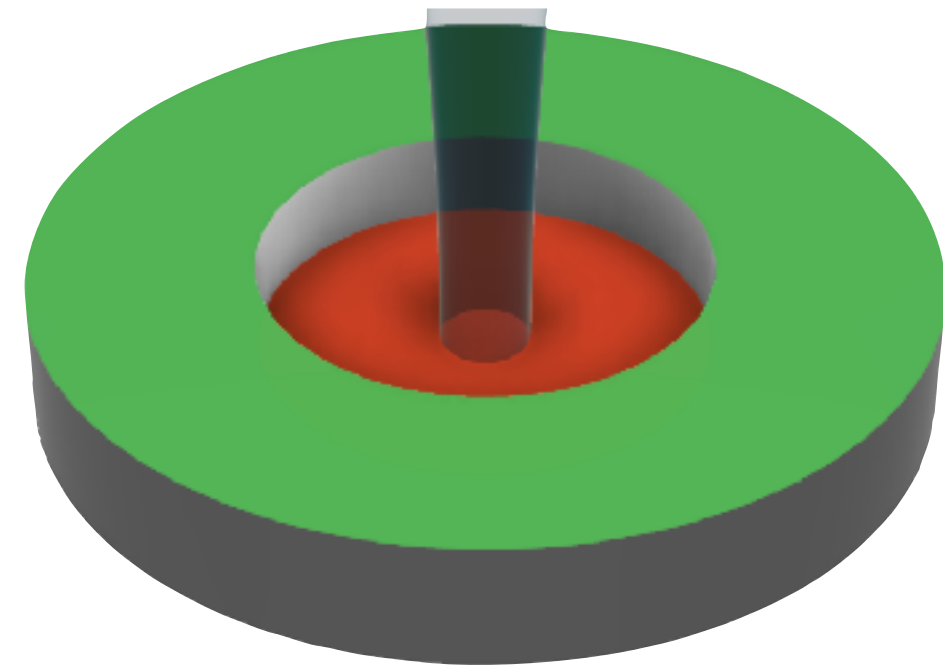
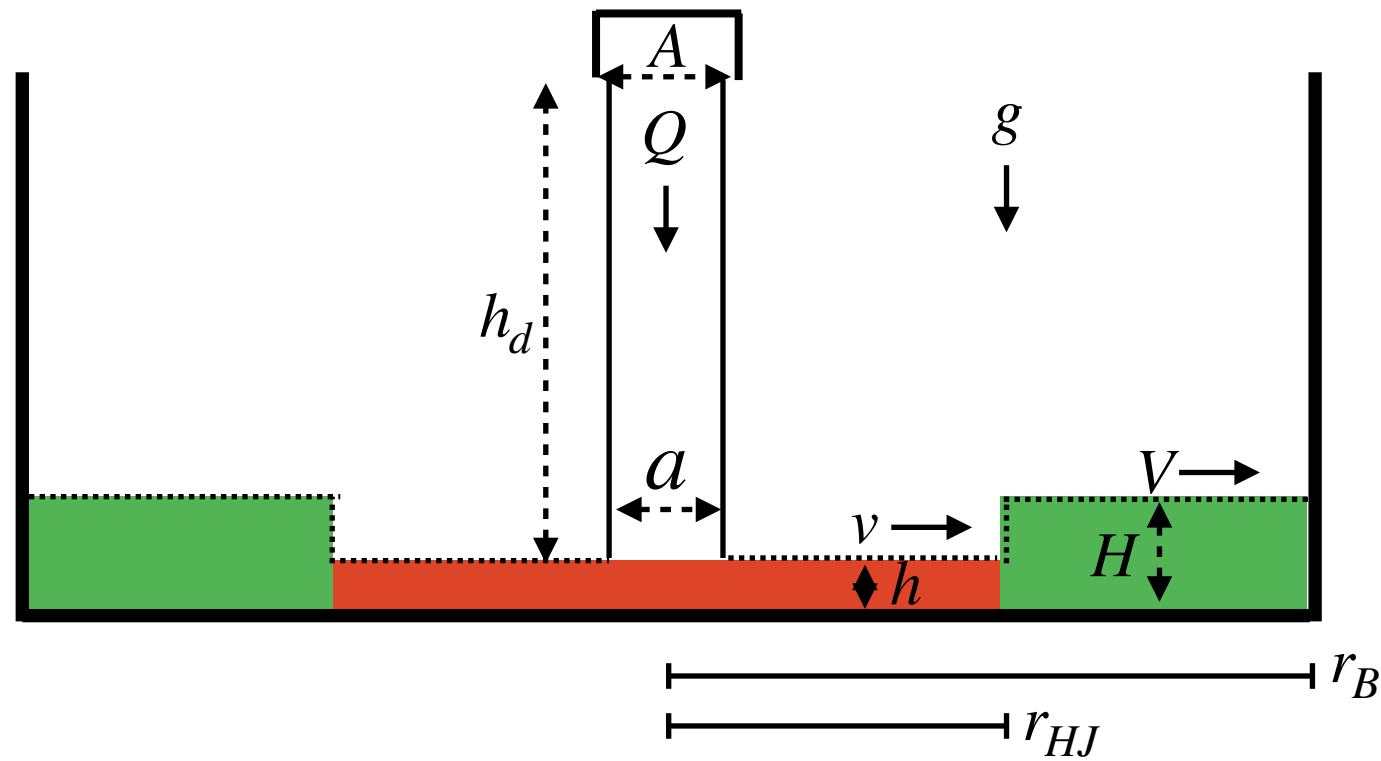


[7] https://www.engineeringtoolbox.com/water-density-specific-weight-d_595.html



Theoretical Approach

Hydraulic jump



$$r_{HJ} = \frac{4Q^2}{\pi^2 a^2 H^2 g}$$

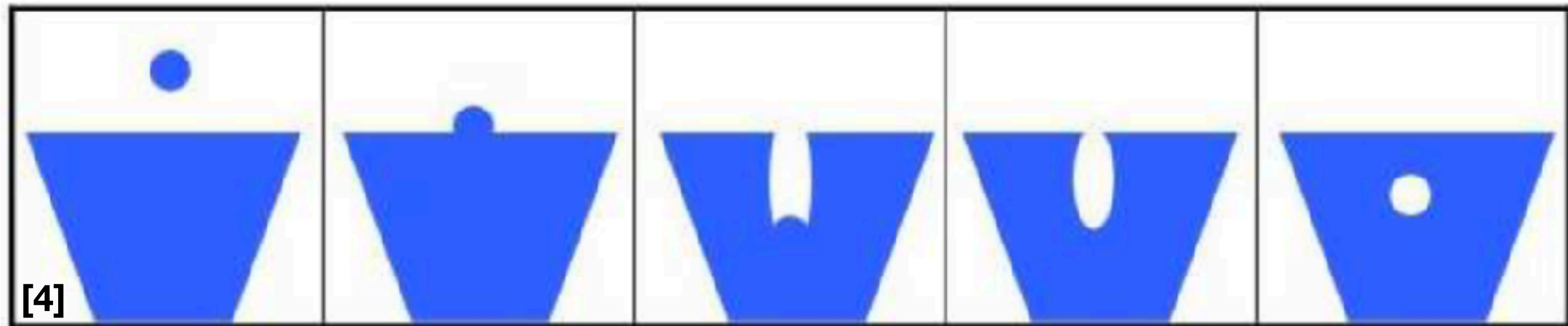
$$Qt = H \cdot r_b^2 - H \cdot r_{HJ}^2 \quad \text{for } h=0$$

$$H = \frac{Qt}{r_b^2 - r_{HJ}^2}$$

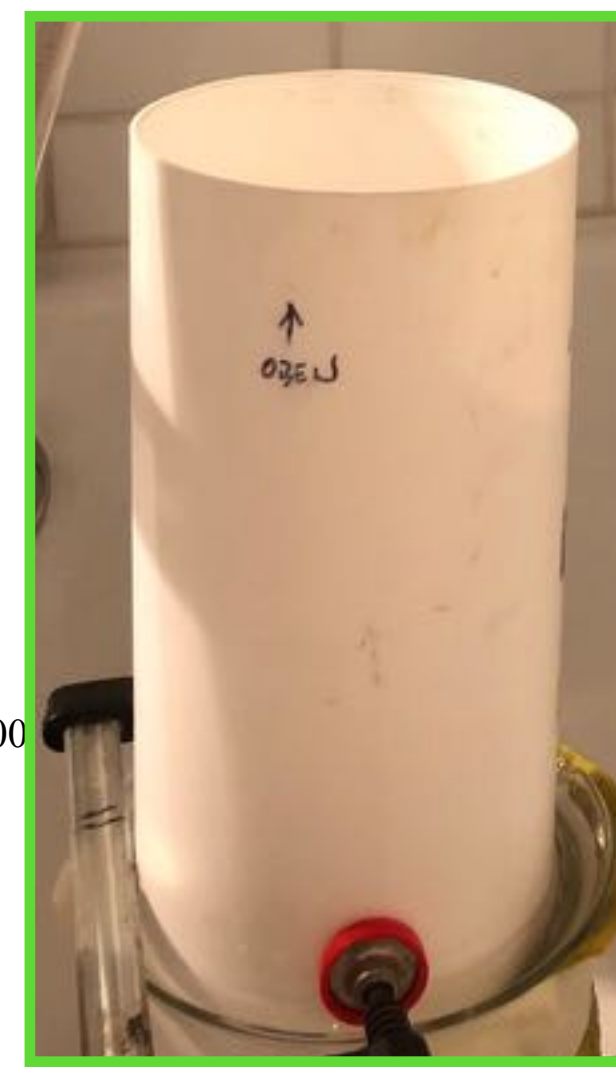
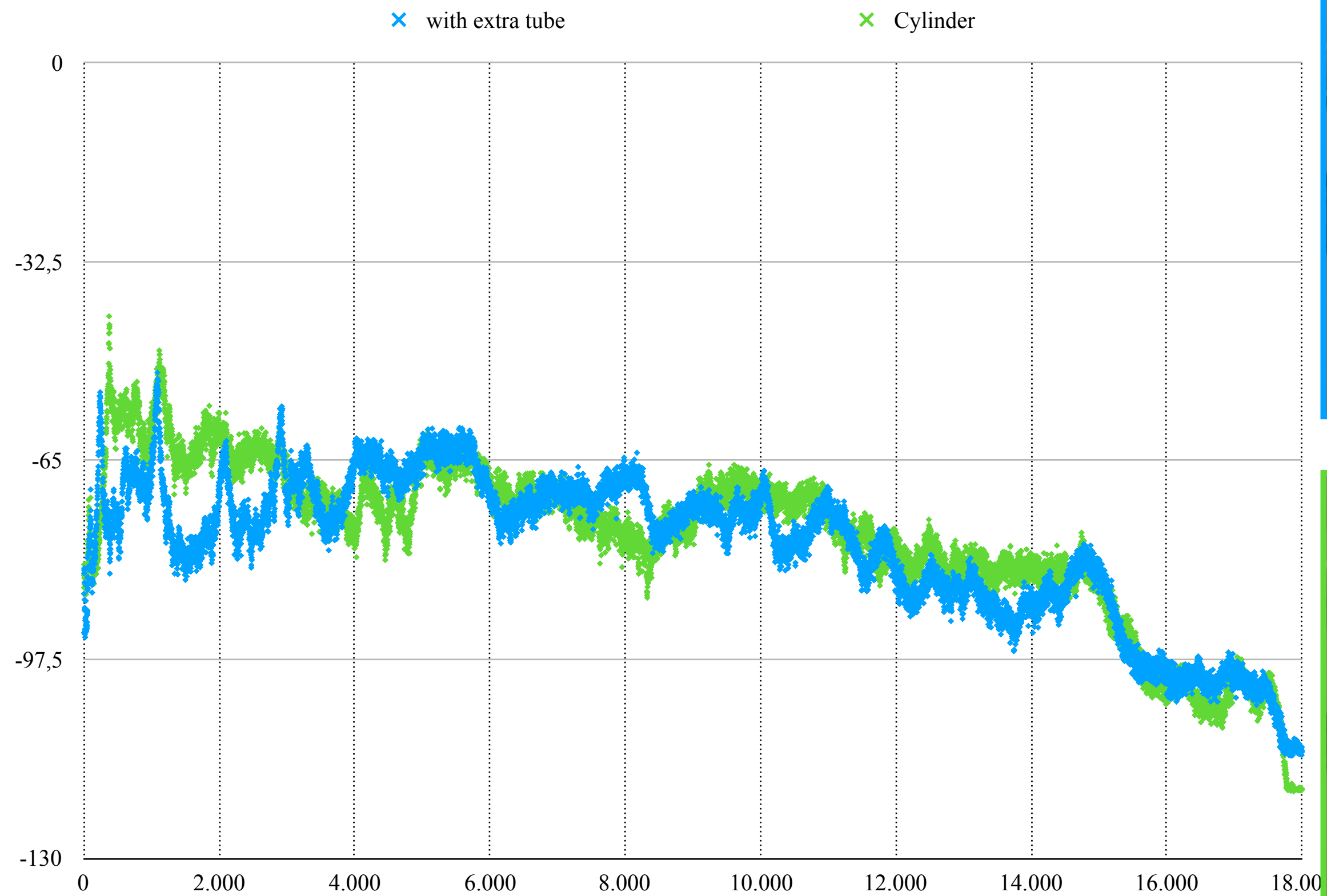
$$r_{HJ} = \frac{4r_b^4 - r_{HJ}^4}{\pi^2 a^2 t^2 g} = 0$$

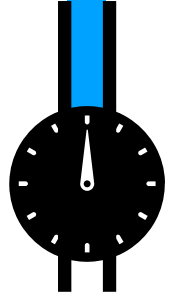
$$\sqrt{\frac{4r_b^4}{\pi^2 a^2 g}} = t = \frac{2r_b^2}{\pi a \sqrt{g}}$$

$$a = \frac{\pi^2 g h_d^{-\frac{1}{4}}}{8Q^2 A^4}$$



[4] Doel, Kees van den. "Physically based models for liquid sounds." (2004)
https://smartech.gatech.edu/bitstream/handle/1853/50904/vandenDoel2_004.pdf





$$Q = 10^{-5} \frac{m^3}{s}$$

$$V = \pi r_B^2 \cdot h_B = 305 \cdot 10^{-6} m^3$$

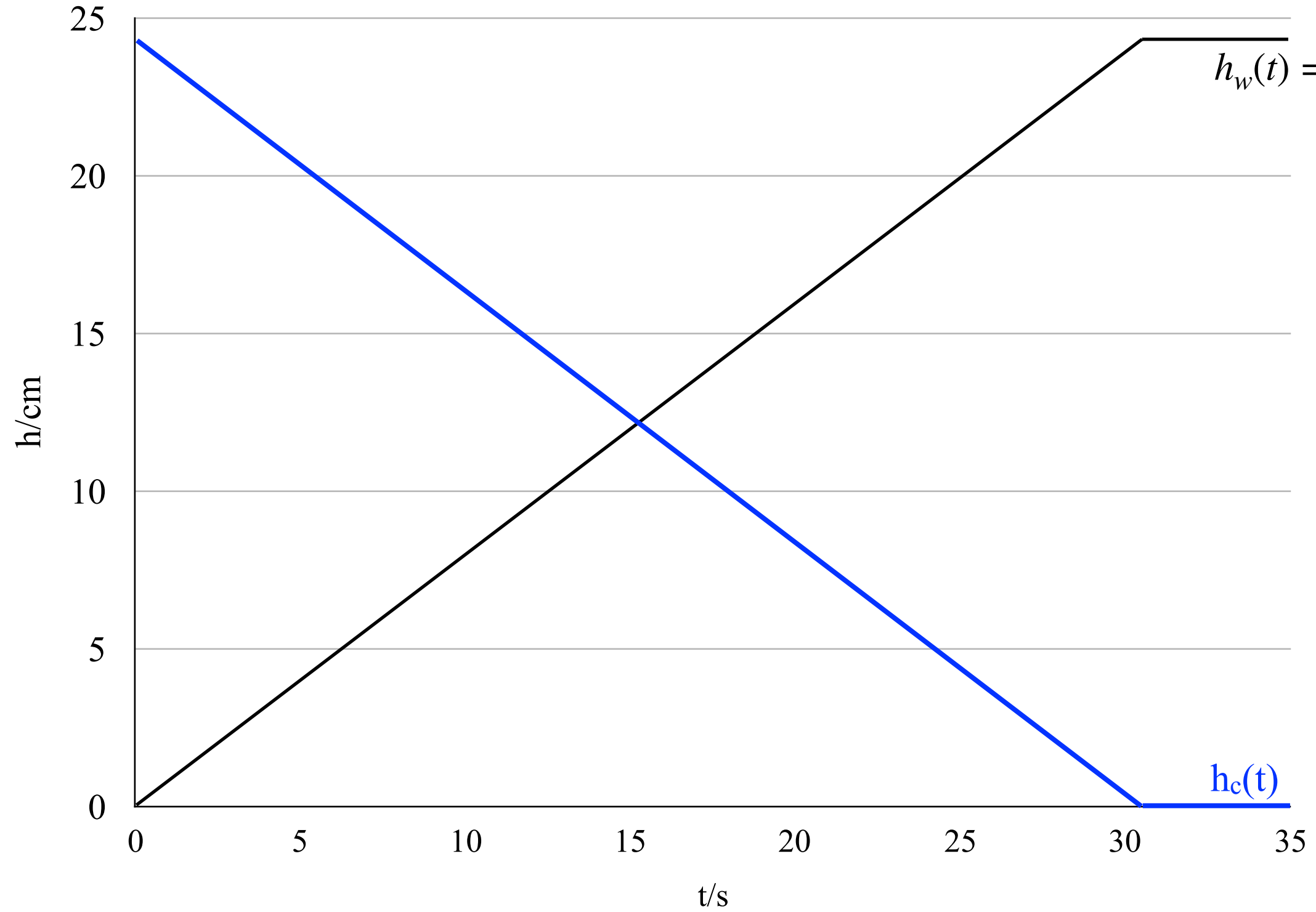
$$t_{full} = \frac{V}{Q} = 30.5 s$$

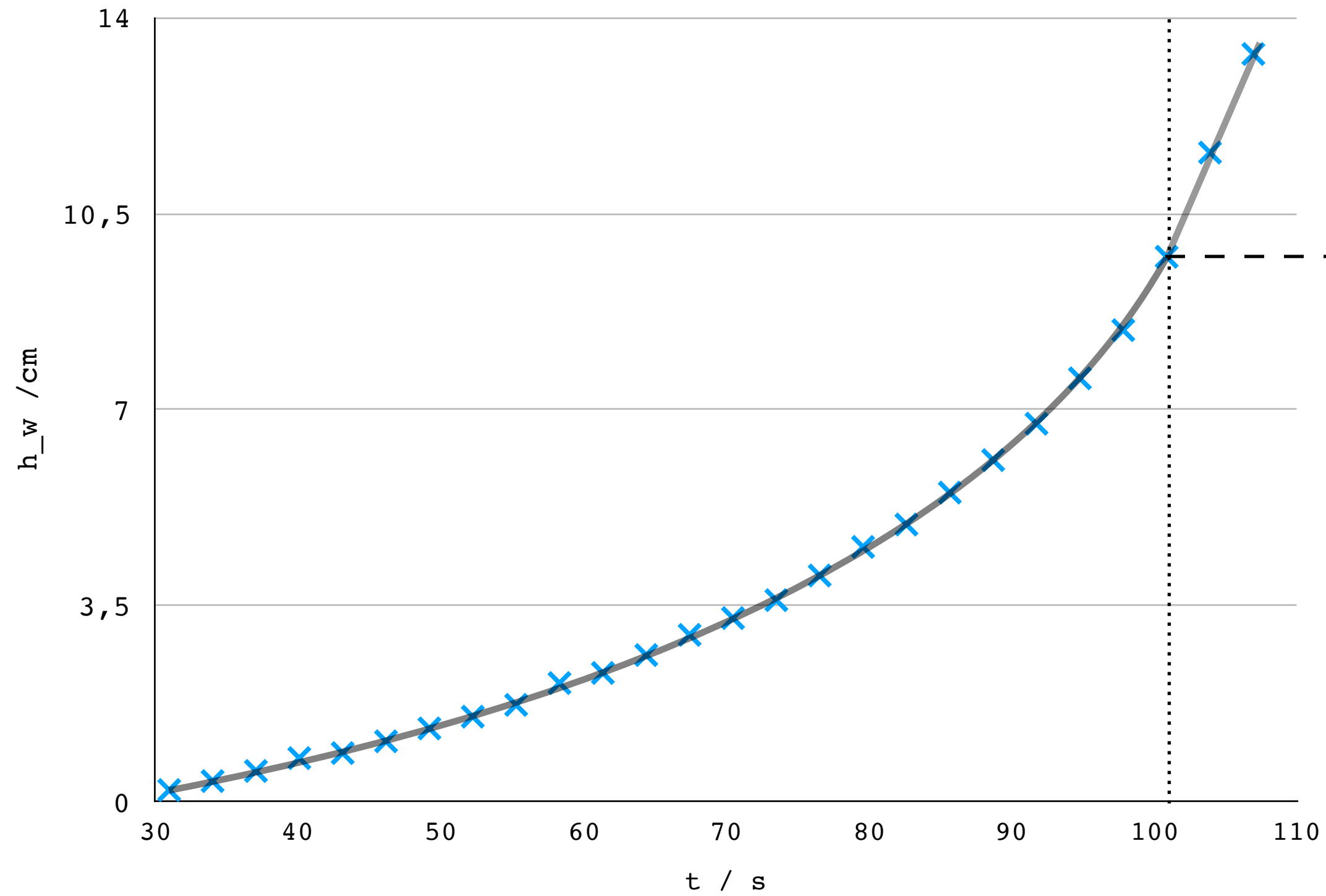
$$f_0 = \frac{c}{4h_c(t)}$$

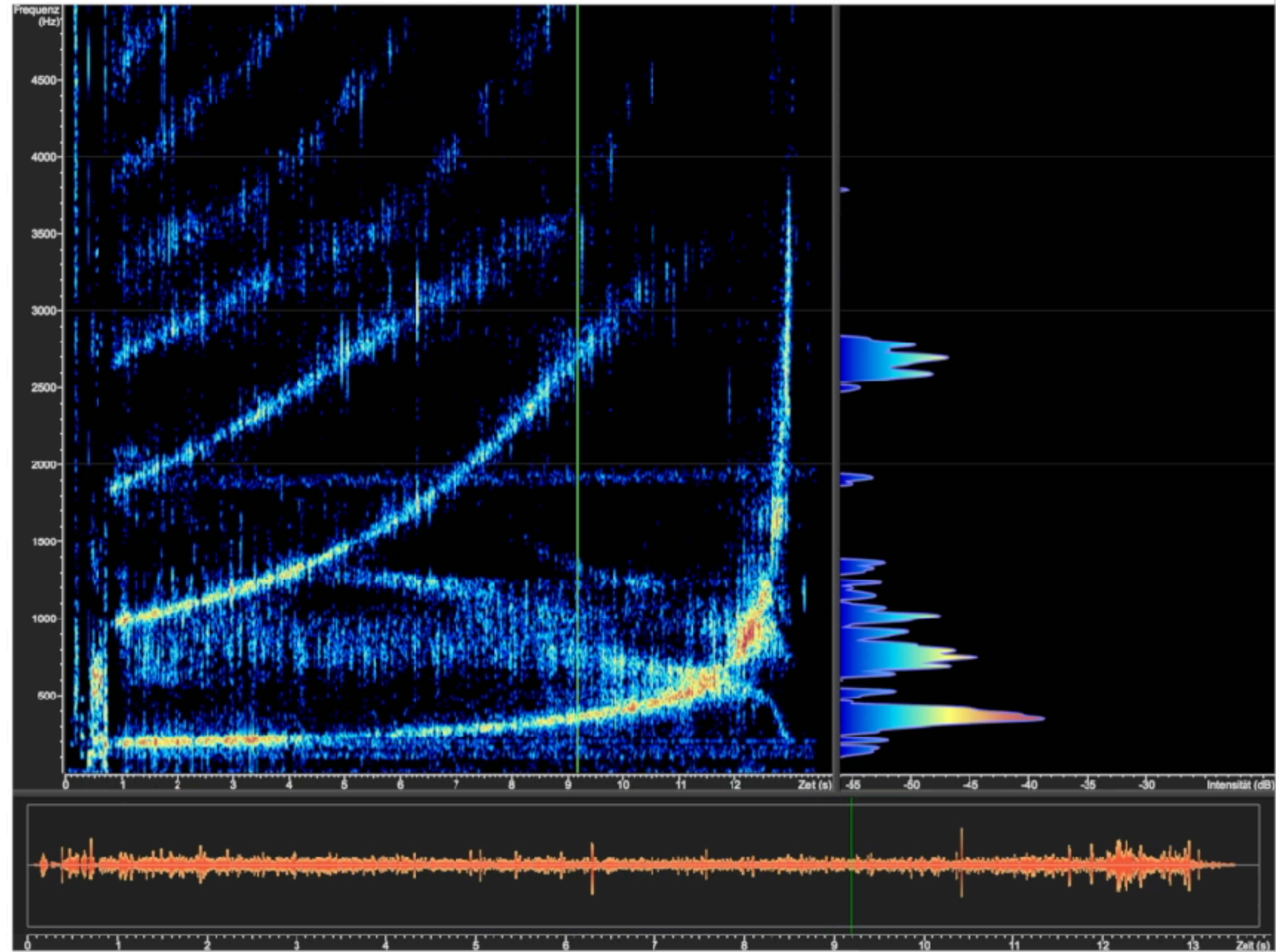
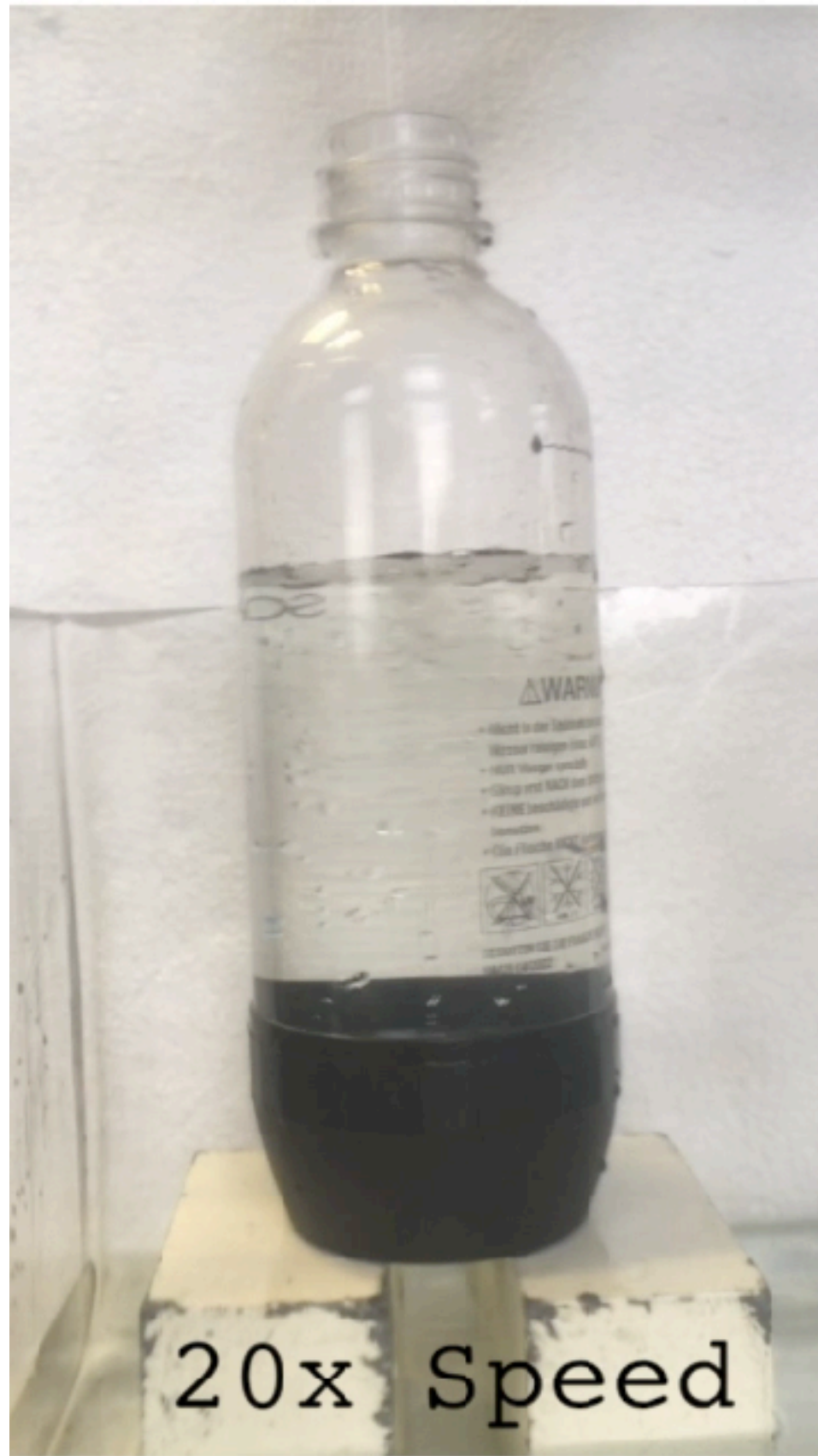


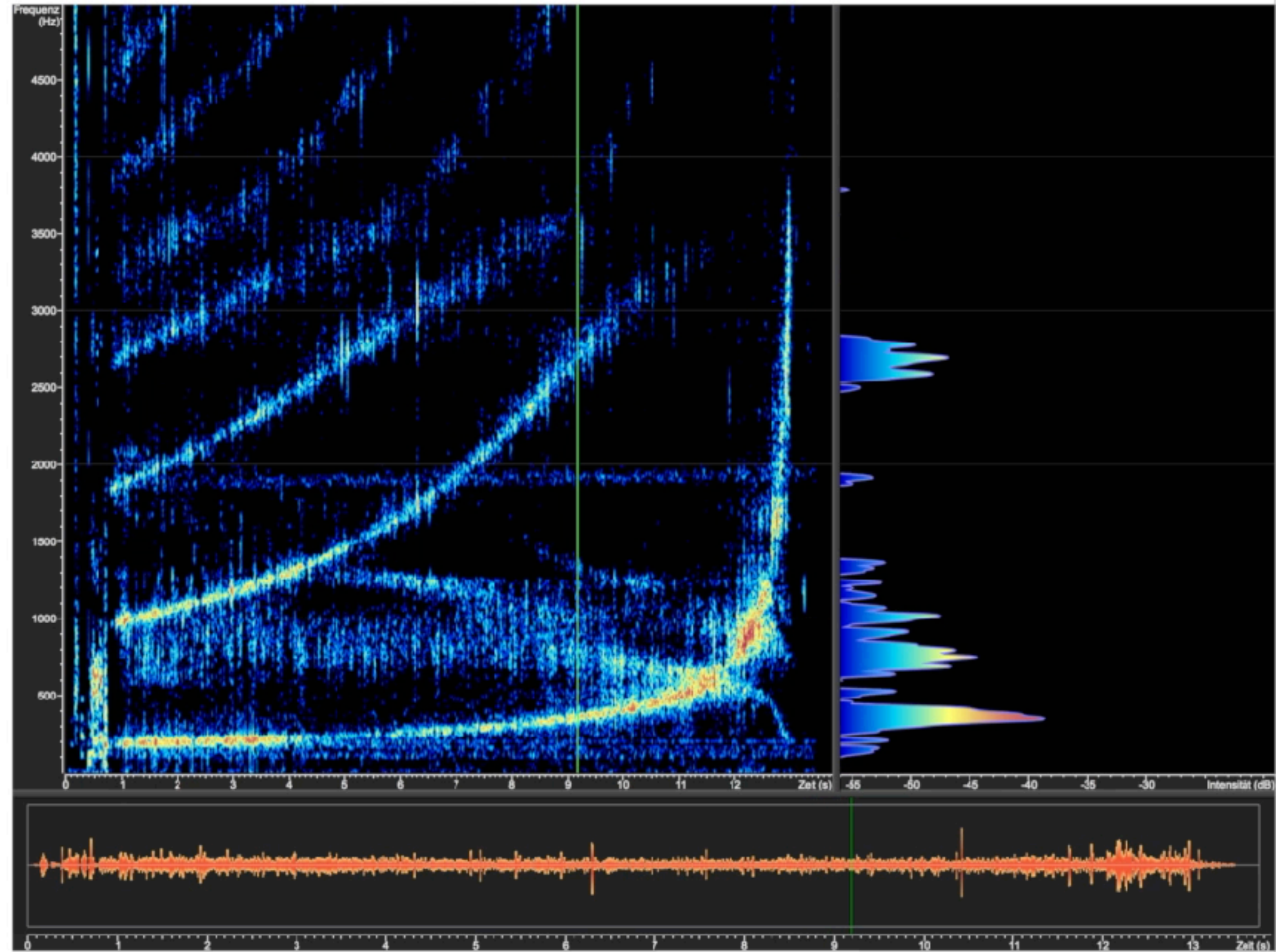
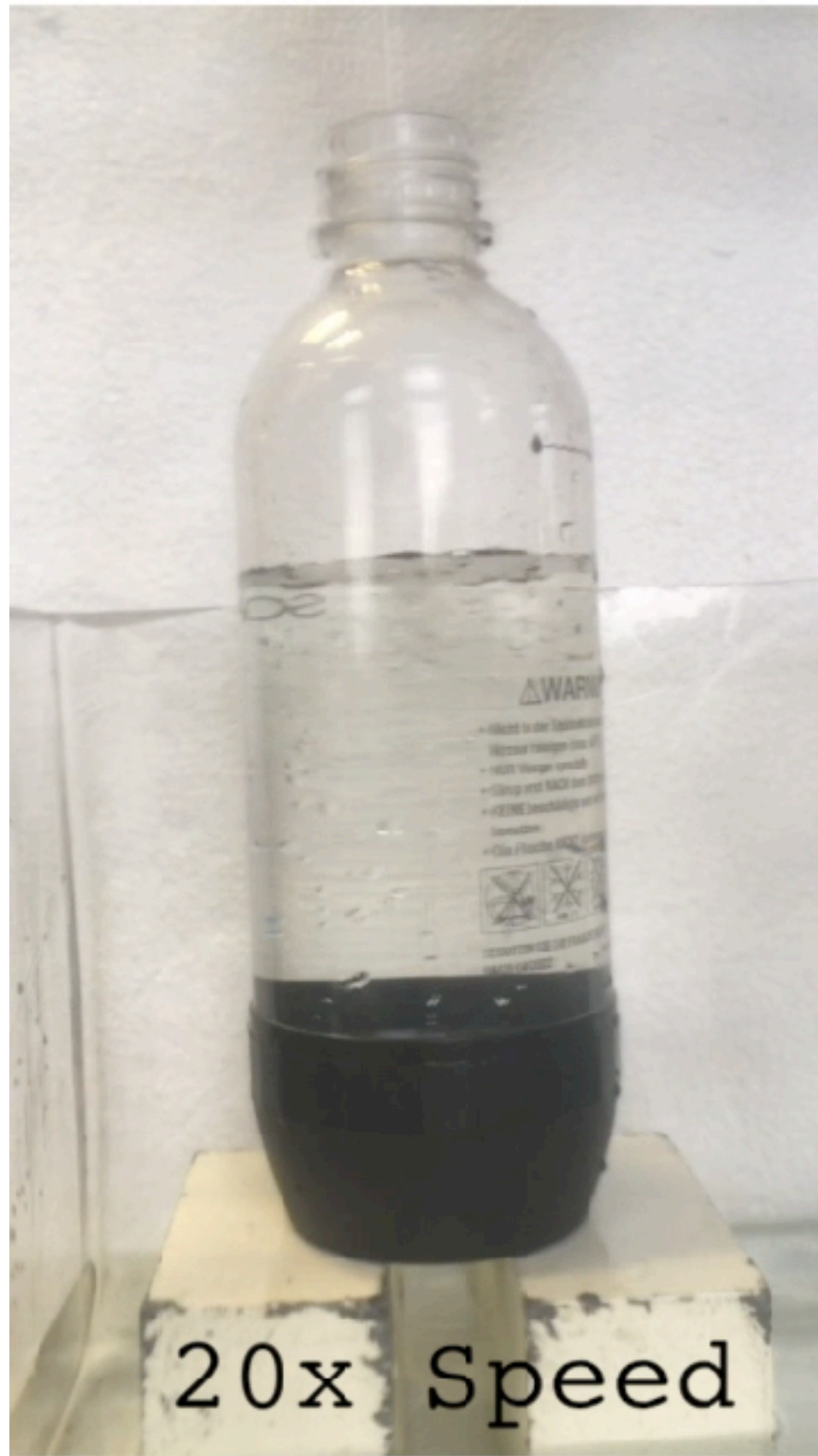
$r_B = 2cm$

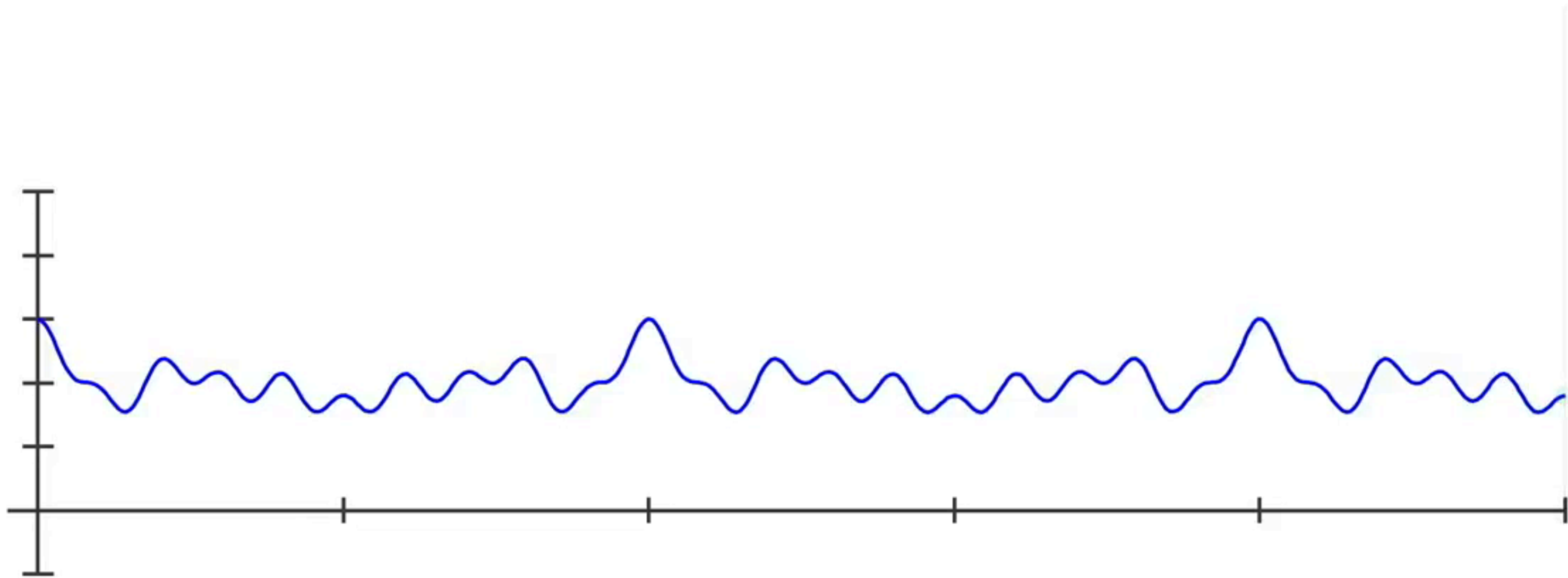
$h_B = 24.3cm$

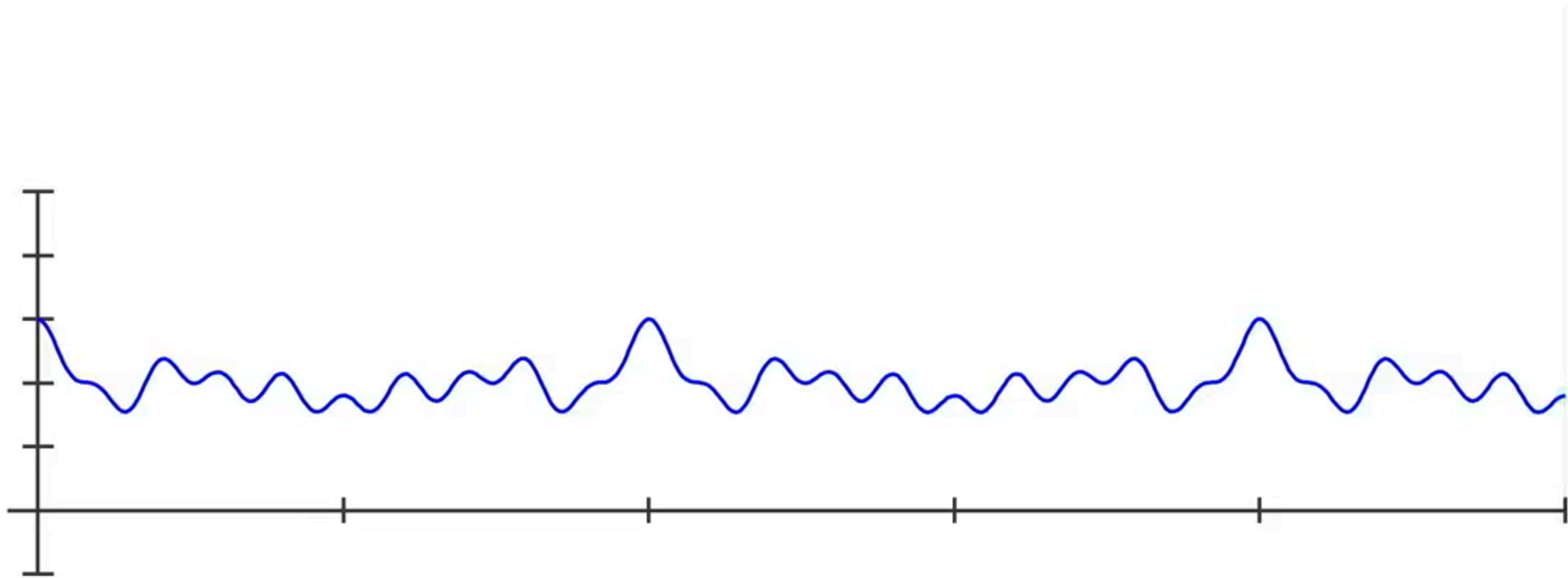


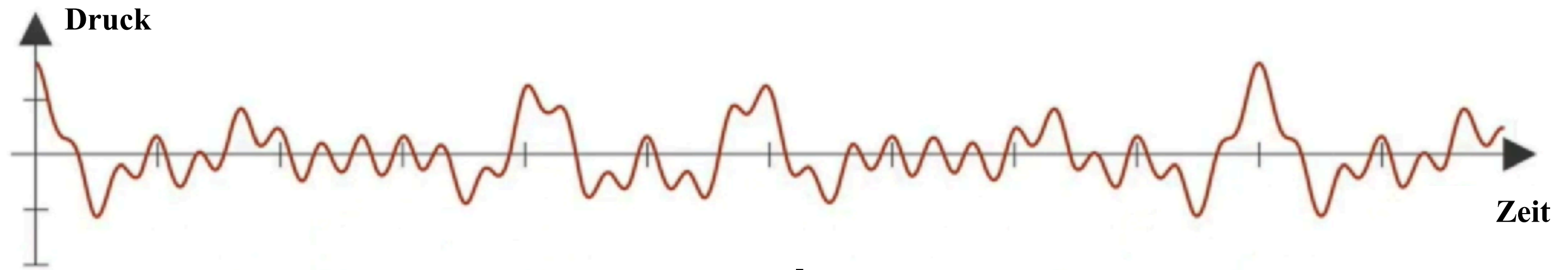












**Fourier
Transformation**

